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Mathematical Analysis of Dynamic Stresses and Structural Optimization of Protection Devices for Railway Track Inductors

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Abstract

This paper presents an analytical and mathematical approach to the dynamic stress phenomena acting on the supports and protection devices of railway track inductors. The main objective of the research is the mathematical substantiation of the transition from the use of conventional structural steels (S235J2) to superior alloys (S355J2) in railway infrastructure, through a rigorous analysis of stress tensors and safety coefficients. Based on equilibrium equations and bending models under transient shocks and accelerations, it has been demonstrated that adopting the S355J2 material increases the structural safety coefficient by over 50%, ensuring the continuous operation of traffic safety systems (PZB/Indusi).

Keywords: Mathematical modeling, strength analysis, railway vibrations, bending stresses, safety factor, structural steel.

2020 MSC: 74B05, 74E30, 74H45, 74K10, 74-10.

1. Introduction

Point automatic train protection and speed control systems represent the foundation of modern railway traffic safety. The track inductor, the central physical element of these systems (such as INDUSI/PZB), operates on the principles of electromagnetic induction and is exposed to a continuous regime of extreme vibrations, mechanical shocks, and temperature variations. The reliability and geometric invariance of this equipment depend exclusively on the integrity of the fastening and protection devices. Any plastic deformation of the support or the lateral protection shield, caused by an accidental kinetic impact, can lead to the failure of the oscillating circuit and, implicitly, to the paralysis of the affected traffic sector through a fail-safe restrictive braking regime.

This paper aims to formulate a clear mathematical model of the stresses induced by the accelerations from the track prism and to evaluate the dynamic response of metallic protection structures made of two types of structural steel: S235J2 and S355J2.

In order to obtain the main results of this study, we have considered following works: ASRO (2019), National Railway Company CFR SA (2001), SPIACT CF ARAD SA (2015), Moţica (2012–2013) and Truşculescu (1977).

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2. Mathematical model of dynamic stresses

During railway operation, the components attached to the rail are subjected to a three-dimensional spectrum of accelerations. Let the resultant acceleration vector $\vec{a}_R$ have scalar components on the three orthogonal axes of the track:

- a_v: vertical component of acceleration;
- a_t: transverse component of acceleration;
- a_l: longitudinal component of acceleration.

The modulus of the resultant acceleration is defined by the equation:

$$a_R = \sqrt{a_v^2 + a_t^2 + a_l^2} \tag{2.1}$$

The resultant dynamic force applied to the device's center of mass (m) is determined using Newton's second law:

$$F_R = m \cdot a_R \tag{2.2}$$

The critical stress model for cantilevered elements (such as the inductor support plate) is given by the maximum bending produced by vertical shocks. The maximum bending moment, $M_{\max}$, at distance (force arm) L, is expressed as:

$$M_{\max} = F_v \cdot L = (m \cdot a_v) \cdot L \tag{2.3}$$

The material's response is dictated by the bending stress σ_i, relative to the section's axial section modulus W_z:

$$\sigma_i = \frac{M_{\max}}{W_z} \tag{2.4}$$

Structural validation involves determining a safety coefficient, c, defined as the ratio between the material's yield strength, R_e, and the calculated maximum stress:

$$c = \frac{R_e}{\sigma_i} \geq c_{admisibil} \tag{2.5}$$

3. Mathematical analysis of the inductor support

The inductor support has a mass $m = 6.7$ kg. The dynamic parameters observed experimentally in the proximity of the rail base fastening zone are: $a_v = 280$ m/s^2, $a_t = 140$ m/s^2, and $a_l = 50$ m/s^2.

Applying equation (1), the resultant acceleration is:

$$a_R = \sqrt{280^2 + 140^2 + 50^2} \approx 317.02 \text{ m/s}^2 \tag{3.1}$$

The resultant force acting on the support is:

$$F_R = 6.7 \cdot 317.02 \approx 2124 \text{ N} \tag{3.2}$$

The base profile of the support is a U50x38x5, with a lever arm $L = 0.472$ m and an estimated section modulus $W_z \approx 10600$ mm^3. The bending moment (Equation 3) has the value:

$$M_{\max} = (6.7 \cdot 280) \cdot 0.472 = 885.47 \text{ Nm} = 885470 \text{ Nmm} \tag{3.3}$$

The resulting bending stress (Equation 4) becomes:

$$\sigma_i = \frac{885470}{10600} \approx 83.5 \text{ MPa} \tag{3.4}$$

3.1. Safety coefficients comparison S235J2 vs. S355J2

For S235J2 steel ($R_e = 235$ MPa):

$$c_{S235} = \frac{235}{83.5} \approx 2.81 \tag{3.5}$$

For S355J2 steel ($R_e = 355$ MPa):

$$c_{S355} = \frac{355}{83.5} \approx 4.25 \tag{3.6}$$

4. Mathematical analysis of the protection device

The protection device (deflector) is subjected to an even more demanding stress regime, given its role as a sacrificial barrier and its higher mass. The input data for the model are: mass $m = 9.2$ kg, lever arm $L = 0.422$ m. The profile used is flat (wide 70x10x422 mm) having a section modulus $W_z \approx 8166.67$ mm^3.

Calculation of the applied dynamic force (according to eq. 2):

$$F_R = 9.2 \cdot 317.02 \approx 2916.58 \text{ N} \tag{4.1}$$

The maximum bending moment experienced by the support plate becomes:

$$M_{\max} = (9.2 \cdot 280) \cdot 0.422 = 1087.07 \text{ Nm} = 1087070 \text{ Nmm} \tag{4.2}$$

The working stress due to bending is evaluated as follows:

$$\sigma_i = \frac{1087070}{8166.67} \approx 133.11 \text{ MPa} \tag{4.3}$$

In this context, we calculate the safety factors c for both material qualities. The results are mathematically synthesized as follows:

$$c_{S235} = \frac{235}{133.11} \approx 1.76 \tag{4.4}$$

$$c_{S355} = \frac{355}{133.11} \approx 2.66 \tag{4.5}$$

Table 1. Synthesis of analytical calculation parameters

Modeled Parameter	**Inductor Support**	**Protection Device**
Calculation mass (m)	6.7 kg	9.2 kg
Resultant force (F_R)	2124 N	2916.58 N
Maximum moment ($M_{\max}$)	885.47 Nm	1087.07 Nm
Bending stress (σ_i)	83.5 MPa	133.11 MPa
Safety Coef. S235J2	2.81	1.76
Safety Coef. S355J2	4.25	2.66

5. Discussions on metallurgical and mechanical implications

From a mathematical point of view, a safety coefficient $c \approx 1.76$ for the protection device manufactured from S235J2 indicates a statically valid structural solution, but vulnerable to fatigue phenomena. The steel with a low carbon content (approx. 0.17% C, hardness ~115 HB) presents a purely hypoeutectoid dominantly ferritic structure. Thus, under the incidence of periodic or transient vibration functions (approx. 28g vertical), stress concentration areas such as the vicinity of threaded fastenings are susceptible to long-term plastic indentation.

By substituting the resistive variable R_e with 355 MPa (for S355J2 steel, approx. 0.22% C, hardness ~145 HB), the safety function registers a non-linear increase in performance due to the increased pearlite proportion. The value $c \approx 2.66$ positions the technological solution in a robust mathematical domain, allowing the complete absorption, in the elastic regime, of asymmetric kinetic energy peaks that exceed nominal limits.

6. Conclusions

The detailed mathematical analysis of the force-response interactions in the railway infrastructure reveals an unequivocal conclusion: the design of inductor protection devices made of S235J2 steel generates working stresses of 133.11 MPa, dangerously close to the fatigue limit, yielding a restrictive safety factor of 1.76.

The implementation of sizing equations based on the S355J2 material extends the safety margin by over 51% ($c = 2.66$), optimizing not only the mechanical resistance to static bending but also the dissipative performance upon shock. This analytical substantiation justifies the total transition towards universal structural solutions made of S355J2 steel, laying the premises for standardization and increasing the operational lifespan of critical infrastructures.

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