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Mathematical Modeling, Structural Resistance Calculation, and Technological Itinerary for Manufacturing the Y25 Bogie Frame

Paul Ilie Vărşendan[a,*]

[a] *Petroleum-Gas University of Ploieşti, Romania*

Abstract

The bogie, serving as the essential running gear for railway vehicles, is a complex welded construction that demands both a precise manufacturing technology and a highly rigorous mathematical validation. While the physical fabrication involves intricate stages of part machining, robotic welding, and assembly, the underlying safety and structural integrity rely entirely on advanced mathematical modeling and computational resistance calculations. This paper outlines the technological itinerary for manufacturing the bogie frame—encompassing the center pivot bolster, side frames, and side bearers—while heavily emphasizing the analytical methodology required to compute resistance, load cases, and dynamic forces. By translating physical phenomena—such as track twist, lateral centrifugal forces, and longitudinal braking impacts—into a comprehensive set of computational algorithms (including Hooke's Law in three dimensions and the von Mises yield criterion), this work bridges mechanical engineering and computer-aided structural analysis. The resulting mathematical framework is highly optimized for finite element analysis (FEA) applications in computer science, facilitating algorithmic optimization of the bogie frame's mass and structural resilience.

Keywords: Technological itinerary, bogie frame, finite element analysis, mathematical modeling, von Mises criterion.

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1. Introduction

The bogie is a fundamental railway vehicle component, acting as the primary running gear that directly interfaces with the railway track. Structurally, it consists of a complex metal skeleton known as the bogie frame (chassis), alongside brake rigging, suspension modules, and the running gear itself. Designed for both passenger and freight transport, the bogie must sustain immense dynamic loads while maintaining kinematic stability.

In freight wagon applications, such as the standard Y25 bogie, a single suspension stage comprising helical springs is typically utilized, contrasting with the multi-stage suspensions found in passenger bogies. The bogie frame itself is an assembly of several critical sub-components: the side frames, the center pivot bolster, side bearers, and brake rigging suspension brackets.

The primary objective of this study is twofold: first, to present the optimized technological itinerary for physical manufacturing; and second, to detail the mathematical modeling and resistance calculations

*Corresponding author
E-mail address: `paul_varsendan@yahoo.com` (Paul Ilie Vărşendan)

that dictate the structural parameters of the frame. This mathematical foundation is crucial for developing robust computer algorithms capable of simulating rail dynamics and ensuring compliance with European standards (such as EN 13749).

The ideas in this study were shaped and materialized by consulting the works Burada *et al.* (1980), Micşa (1981), Nanu (1983) and Sebeşan & Copaci (2008).

2. Technological itinerary for manufacturing

Before initiating the physical production of a bogie frame, several preliminary stages are executed, including advanced computer-aided design (CAD), study of technical drawings, execution of work instructions, and the formulation of precise welding plans. The manufacturing itinerary systematically progresses through cutting, verification of dimensional tolerances (e.g., EN 2768), and specialized machining operations (chamfering, drilling).

2.1. Component machining and assembly

Cutting operations are highly automated, utilizing CNC laser, plasma, or oxy-gas cutting depending on sheet thickness. Once parts are cut and verified, the assembly process begins with the center pivot bolster—the central structural node of the bogie. The lower flange, upper flange, web, and inner stiffeners undergo bending and chamfering before being mounted into specialized rotary devices. The elements are tack-welded, and following dimensional verification, robotic welding units execute the final structural seams.

A similar highly-controlled itinerary is applied to the side frames. Following tack-welding and robotic seam execution, stiffeners are mounted and welded. Non-destructive testing (NDT), utilizing penetrant liquids, magnetic powders, X-ray (RX), or ultrasound (US), guarantees the molecular integrity of the welds.

The final integration involves mounting the pivot bolster and side frames into a rigid device. Torsion is strictly eliminated (maximum admitted tolerance: 2 mm). Subsequently, four side bearers (three type I and one type II) are welded to the lower flange of the side frames. Final machining of the side bearer dimensions is performed on large-scale CNC milling centers (e.g., UNISIGN), followed by metal grit sandblasting (grade 2.5) and coating with 2KEP bi-component primer and RAL 9005 paint.

3. Mathematical modeling and computational parameters

While the physical manufacturing ensures dimensional accuracy, the survivability of the frame depends entirely on mathematical load modeling. The system of forces acting upon the Y25 bogie frame constitutes a three-dimensional computational problem governed by vehicle mass, track imperfections, and kinematic accelerations. For algorithmic processing in FEA software, these are classified into exceptional loads (verifying yield strength limits) and normal service loads (evaluating fatigue life cycles).

3.1. Technical parameters for algorithmic input

The mathematical model relies on a strict set of boundary conditions and reference constants defined by the Y25 bogie specifications:

- Gauge: 1435 mm (defines the transversal force moment arm).
- Wheel base ($2a^+$): 1800 mm (essential for calculating longitudinal bending moments).
- Maximum axle load: 22.5 t (the fundamental vertical loading constraint).

- Tare weight (m^+): 4450 kg ± 5% (the unsprung mass used in static equations).
- Maximum running speed: 120 km/h (determines dynamic excitation frequencies).
- Maximum cant deficiency: 130 mm (generates lateral uncompensated acceleration of ~0.8–1.05 m/s^2).

4. Analytical methodology for load cases

This section details the explicit mathematical formulas utilized to compute the primary tensor fields acting on the structure. These formulations are directly translatable into matrix operations for structural simulation software.

4.1. Definition of reference masses and vertical loads

Let M_v be the total mass of a standard freight wagon equipped with two Y25 bogies, loaded to its maximum permitted capacity. Given an axle load of 22.5 t, the total operational mass ($M_{v,total}$) is computed as:

$$M_{v,total} = 4 \times 22.5 \text{ t} = 90 \text{ t} \tag{4.1}$$

The net static vertical load (F_z) applied to the center pivot excludes the unsprung wheelsets and components resting directly on the axles. According to equation C.19 from EN 13749, this is formulated as:

$$F_z = \frac{(M_{v,total} - 2m^+) \cdot g}{2} \tag{4.2}$$

Substituting the predefined numerical parameters (where $g \approx 9.81$ m/s^2):

$$F_z = \frac{(90000 \text{ kg} - 2 \times 4450 \text{ kg}) \cdot 9.81}{2} \approx 397.8 \text{ kN} \tag{4.3}$$

Under perfect track alignment, this force distributes symmetrically. However, to mathematically model exceptional dynamic shocks, a global safety scalar (typically 1.4 to 1.5) is multiplied into the matrix, resulting in a maximum localized vertical force ($F_{z,max}$) of approximately 298.3 kN per side frame.

4.2. Transversal loading and the Prud'homme limit

Lateral forces in curve negotiations are critical computational vectors. The exceptional transversal force ($F_{y,max}$) is mathematically constrained by the Prud'homme Limit, representing the threshold before permanent ballast displacement occurs. The standard formula (EN 13749 C.18) is computed as a function of the nominal static axle load ($P_0 = 220.7$ kN):

$$F_{y,max} = 10 + \frac{P_0}{3} \text{ (kN)} \tag{4.4}$$

$$F_{y,max} = 10 + \frac{220.7}{3} \approx 177.5 \text{ kN} \tag{4.5}$$

This immense lateral vector necessitates high torsional stiffness in the side frames. Algorithmic simulations must apply this vector cyclically to determine fatigue resilience.

4.3. Longitudinal dynamics and braking tensors

Longitudinal efforts arise from traction, buffing (impacts), and braking. Braking vectors ($F_{x,brake}$) are mathematically limited by the maximum wheel-rail adhesion coefficient ($\mu \approx 0.3$–0.4):

$$F_{x,brake} = \mu \cdot \frac{F_z}{4} \tag{4.6}$$

These asymmetrical forces induce a computational "lozenging" effect, distorting the rectangular matrix of the bogie frame into a rhombus configuration, heavily stressing the welded joints.

4.4. Critical kinematic loading: track twist

Derailment prevention dictates a strict mathematical relationship between torsional stiffness and flexibility. The EN 13749 standard imposes a computational simulation of a track twist irregularity of 0.5%. The kinematic vertical displacement (Δz) enforced at a single axle box, assuming the other three define a stable plane, is the product of the wheelbase ($2a^+$) and the twist gradient:

$$\Delta z = 2a^+ \cdot 0.005 = 1800 \text{ mm} \cdot 0.005 = 9 \text{ mm} \tag{4.7}$$

This 9 mm geometric translation represents the most severe computational scenario for the central pivot welds, often being the primary source of structural singularity (cracking) during mathematical modeling without primary suspension efficiency.

5. Computational mechanics for finite element analysis

Modern structural validation requires translating these one-dimensional mathematical load cases into complex, three-dimensional mesh algorithms. This superposition is mathematically controlled using kinematic coefficients: roll ($\alpha = 0.1 \div 0.2$) and bounce ($\beta = 0.2 \div 0.3$).

5.1. 3D Hooke's law and elastic moduli

Finite element algorithms rely on the generalized Hooke's Law to compute material strain. For stresses applied across three orthogonal axes (x, y, z), the specific linear deformation (ε_x) is computed utilizing Young's Modulus ($E = 210$ GPa) and Poisson's Ratio ($\nu = 0.3$):

$$\varepsilon_x = \frac{1}{E} \cdot \left[\sigma_x - \nu \cdot \left(\sigma_y + \sigma_z\right)\right] \tag{5.1}$$

Furthermore, the Transverse Elastic Modulus (Shear Modulus, G), which dictates the matrix resistance to the 9 mm track twist, is algorithmically derived:

$$G = \frac{E}{2 \cdot (1 + \nu)} \tag{5.2}$$

$$G = \frac{210 \cdot 10^9}{2 \cdot (1 + 0.3)} \approx 80.76 \text{ GPa} \tag{5.3}$$

This scalar value of 80.76 GPa is a fundamental input for the stiffness matrices generated during computational simulations.

5.2. von Mises yield criterion

To determine structural failure within the computer model, the algorithm calculates the von Mises Equivalent Stress (σ_{vM}). This formula amalgamates all tensile (σ) and shear (τ) forces at a given mesh node into a single scalar value, which is then compared against the material's yield strength ($R_e = 355$ MPa for S355 steel):

$$\sigma_{vM} = \frac{1}{\sqrt{2}} \cdot \sqrt{\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)} \tag{5.4}$$

The algorithm evaluates safety by ensuring that $\sigma_{vM} \leq (R_e/c_s)$, where c_s is the designated safety factor. Stresses exceeding this threshold indicate plastic deformation, requiring automated geometrical thickening of the simulated CAD model.

5.3. Volumetric mass integration and inertial vectors

Inertial loads during severe impacts (simulated up to 5g) are mathematically dependent on the exact integration of the material density ($\rho = 7850$ kg/m^3) over the volumetric mesh (V):

$$M = \int_V \rho \, dV \tag{5.5}$$

According to Newton's second law, an empty frame mass of 1500 kg subjected to a 3g longitudinal acceleration generates a self-induced inertial force (F_x) of:

$$F_x = 1500 \cdot 3 \cdot 9.81 \approx 44.14 \text{ kN} \tag{5.6}$$

This computational result proves that the mass of the steel itself acts as a significant destructive vector during collisions. Finally, the crystalline ductility of the steel is defined algorithmically as an Impact Breaking Energy constraint, mandating the absorption of at least 27 Joules at -20°C (Charpy V-notch parameter) before catastrophic structural bifurcation occurs.

6. Conclusions

The manufacturing of the Y25 bogie frame (mass 4.45 t) is a highly sophisticated process requiring seamless integration between physical metallurgy and computational mathematics. The technological itinerary—spanning laser cutting, robotic welding, and CNC machining—is fundamentally driven by analytical load models. By quantifying physical phenomena into explicit mathematical tensors, such as the Prud'homme limit, kinematic twist displacement, and von Mises stress criteria, engineers can algorithmically validate the structural integrity of the frame. Future research directions in applied computer science should focus on topological optimization algorithms aimed at reducing the bogie's unsprung mass through strategic cutouts and lighter materials, whilst automating the robotic assembly processes to increase industrial productivity.

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