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Mathematical Analysis and Structural Optimization of an Insulated Support for the 25 kv–50 hz Railway Contact Line

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Abstract

This paper presents a structural analysis of an insulated support assembly used in 25 kV–50 Hz railway overhead contact lines. The study is based on the supplied engineering drawings of the stay rod, brace, direct steady arm, reverse fixing device, bar, couplings, fixing plate and contact-wire clamp. The dimensions and materials explicitly readable in the drawings are incorporated into a simplified analytical model for bending, shear and elastic displacement. The reference configuration mainly uses S235/S235JR/FMN35 steels with corrosion protection by hot-dip galvanizing, while CuZn15 and other steels/stainless components are present in the contact clamp and fasteners. AlCu4Mg is investigated as a lightweight alternative for selected structural members. The analysis confirms the substantial mass-saving potential of aluminium alloys, but also the associated loss of stiffness; therefore, a material substitution must be accompanied by geometric redesign. For the $\varnothing 34 \times 4$ mm tubular direct steady arm, with $L = 400$–1600 mm, the section properties are calculated and a conservative $F_d = 90$ N case is used to quantify the sensitivity of bending moment and elastic displacement to length. The proposed methodology combines mass, strength and stiffness criteria and can be extended by finite-element analysis.

Keywords: Railway contact line, strength of materials, structural optimization, finite element analysis.

2020 MSC: 74B05, 74K10, 74P10, 74S05

1. Introduction

Railway electrification at 25 kV–50 Hz requires maintaining a stable position of the contact wire and, implicitly, a controlled geometry of the supporting system.

The insulated support is a structural assembly that transmits to the supporting infrastructure the loads arising from the self-weight of the components, conductors, wind action, ice accretion and pantograph–wire dynamic interaction. In the analyzed documentation, the assembly consists of a series of interdependent components: stay rod, brace, steady arm, bar, fixing devices, couplings, fixing plate and clamp.

The aim of the paper is twofold. First, it seeks to systematize the geometric and material data from the supplied drawings so that they can be used in a reproducible calculation model. Second, it seeks to evaluate the possibility of reducing mass by selectively replacing steel with AlCu4Mg, without compromising stiffness and the support function.

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The supplied technical documentation contains length variants for the stay rod, brace and steady arm. For example, the stay rod length is indicated in the range of 1650–3600 mm, the brace length in the range of 2680–4910 mm, while for the steady arm, the length is indicated between 400 and 1600 mm. These ranges are important because the bending moment increases linearly with the force arm, whereas the elastic displacement of a cantilever increases approximately with the cube of its length.

We have constructed this paper and obtained the main results after consulting following documents: Deutsches Institut für Normung (1991), European Committee for Electrotechnical Standardization (2002), European Committee for Standardization (2005), European Committee for Standardization (2019),European Committee for Electrotechnical Standardization (2020), International Organization for Standardization (2022), International Electrotechnical Commission (2024), and Petroleum-Gas University of Ploieşti (2026).

2. State of the problem and research hypothesis

From a mechanical design perspective, replacing a structural steel with an aluminium alloy is not a simple material substitution. For the same geometry, density decreases substantially, but the elastic modulus is reduced by approximately three times. Consequently, an AlCu4Mg structure may be lighter but may exhibit larger displacements. For components that control the position of the contact wire, the stiffness criterion may become as important as strength.

The research hypothesis is that a significant mass reduction can be achieved by using an aluminium alloy for selected components, provided that the geometry is optimized so that the EI product remains sufficiently high. In this formulation, E is the elastic modulus and I is the second moment of area of the section.

The material values for the S235–AlCu4Mg comparison are taken from the research material included in the initial document: density 7850 kg/m^3 for S235 and 2780 kg/m^3 for AlCu4Mg; elastic modulus 210 GPa and approximately 70–75 GPa, respectively; tensile strength 360–510 MPa for S235 and 450–520 MPa for AlCu4Mg. These values are used as analysis parameters, not as a substitute for certification of the material intended for production.

3. Structural configuration and drawing data

Analysis of the drawings shows that the assembly is not a single part, but a system of elements with different functions. Components subjected predominantly to axial loading and bending must be analyzed together with the joints and fastening elements, because the local stiffness of the connections influences the global response.

Corrosion protection requirements according to SR EN ISO 1461 are also specified for the indicated metallic components, together with general tolerances according to DIN ISO 2768-1-mk. For the contact-wire clamp, the materials differ from those of the main elements: the jaw is made of CuZn15, the pivot is indicated as 12C13, and some elements are made of S235. This differentiation is relevant for any material substitution analysis.

4. Materials and mechanical properties

In the documented configuration, S235/S235JR and FMN35 are associated with the main structural elements and with machined/welded parts. Stainless steel and CuZn15 are present in certain elements. In the optimization analysis, AlCu4Mg is introduced as an alternative material for components where electrical insulation, corrosion, joining and galvanic compatibility allow this solution.

Component	**Geometric data/variants from drawing**	**Material/remarks from drawing**
Stay rod	$L = 1650 \dots 3600$ mm; variants 01–09	D-type rod insulator; S235JR hook; stainless-steel fastening elements; corrosion protection
Brace	$L_{cf} = 2680 \dots 4910$ mm; variants 01–15	Assembly with insulator; galvanized tube; S235JR elements and accessories
Direct steady arm	$L = 400 \dots 1600$ mm; $\varnothing 34 \times 4$ mm tube	Tube; plug; base plate; rivets; joint detail 3×N-89
Reverse straight fixing device	Rectangular tube 40×40×2.5; ∅16×123 hook; $6 \times \varnothing 4 \times 60$ pins	S235JR/subassemblies; galvanizing protection
Bar	$L = 850$ mm (direct) / 1500 mm (reverse), according to the note	Manufacturing/corrosion protection by galvanizing
Eye coupling	Body $109 \times 60 \times 16$; M12×30 bolt	S235/FMN35; indicated mass 0.61 kg for the corresponding assembly
Terminal coupling	Body ∅52; $H \approx 112$ mm	S235/FMN35; indicated mass ≈ 0.51 kg
Fixing plate	$150 \times 90 \times 10$; $\varnothing 12 \times 350$ stainless-steel rods	S235JR/FMN35 + stainless steel; indicated mass ≈ 1.02 kg
1” stay-rod coupling	$265 \times 65 \times 16$; ∅20 hole; functional dimensions 155/200 mm	S235/FMN35; indicated mass ≈ 2.311 kg
Contact-wire clamp main jaw	68×24×21; ∅16×36 pivot; fixing head $21 \times 38 \times 50$	CuZn15, S235, 12C13, M8×30 bolt; mass ≈ 0.412 kg

For the same geometry, the ratio of bending stiffnesses is controlled by the ratio of elastic moduli. If $E_s = 210$ GPa and $E_a = 70$ GPa, then

$$\frac{E_s}{E_a} \approx \frac{210}{70} = 3. \tag{4.1}$$

Thus, to approximately maintain the same bending stiffness, the second moment of area of the aluminium section must be about three times larger than that of the steel section:

$$E_a I_a \approx E_s I_s. \tag{4.2}$$

5. Geometric modeling of the steady arm

For a mathematical verification in which the dimensions from the drawing can be used directly, the direct steady arm is selected because the drawing explicitly specifies the $\varnothing 34 \times 4$ mm tube and a series of lengths $L = 400, 500, \dots, 1600$ mm. For the circular tube, the outer diameter $D = 34$ mm and inner diameter $d = 26$ mm are considered.

For the circular tube:

$$A = \frac{\pi}{4}\left(D^2 - d^2\right). \tag{5.1}$$

$$I = \frac{\pi}{64}\left(D^4 - d^4\right). \tag{5.2}$$

$$W = \frac{I}{D/2}. \tag{5.3}$$

Parameter	**S235**	**AlCu4Mg**	**Al/S235 ratio**	**Significance**
Density ρ [kg/m^3]	7850	2780	0.354	Theoretical mass reduction ≈ 64.6% at the same volume
Modulus E [GPa]	210	70–75	0.33–0.36	The stiffness of the aluminium material is much lower
R_m [MPa]	360–510	450–520	—	Comparable tensile-strength ranges
$R_e/R_{0.2}$ [MPa]	≥ 235	≈ 320–400	—	The heat-treated alloy may have high yield strength
Relative mass at the same volume	1.000	0.354	0.354	Potential for mass reduction

This gives $A = 377.0$ mm^2, $I = 43165$ mm^4 and $W = 2539$ mm^3. For S235, the theoretical linear mass of the tube is 2.96 kg/m, consistent with the table in the drawing, which indicates approximately 1.18 kg for $L = 400$ mm and 4.74 kg for $L = 1600$ mm.

The difference between the calculated mass of the simple tube and the total assembly mass is explained by the plug, base plate, rivets and other component parts. Consequently, the total mass of the steady arm should not be reduced to the mass of the tube.

L [mm]	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600
Tube mass [kg]	1.18	1.48	1.78	2.07	2.37	2.66	2.96	3.26	3.55	3.85	4.14	4.44	4.74

The table above reproduces the tube mass values from the steady-arm drawing. The variation is nearly linear with length, confirming that the section is constant along the tubular portion.

6. Loading model and calculation equations

According to the research data, the Cu-ETP contact wire with a cross-section of 120 mm^2 has a mass of approximately 1.07 kg/m, the average additional load due to rime ice can be approximated at 1.0 kg/m, and the maximum dynamic pantograph–catenary force is $F_d = 90$ N. For wind, a basic dynamic pressure of 91 daN/m^2 without rime ice and 25 daN/m^2 in the presence of rime ice is indicated.

$$q_c = m_c g = 1.07 \cdot 9.81 \approx 10.50 \text{ N/m}. \tag{6.1}$$

$$q_i = 1.00 \cdot 9.81 \approx 9.81 \text{ N/m}. \tag{6.2}$$

$$q_v = q_c + q_i \approx 20.31 \text{ N/m}. \tag{6.3}$$

$$F_w = p_w A_p. \tag{6.4}$$

In a preliminary model, the distributed load can be converted into a resultant over length L, while the dynamic force can be introduced as a concentrated force. For a simplified cantilever with uniform load q and force F at the end, the maximum moment at the support is:

$$M_{\max} = \frac{qL^2}{2} + FL. \tag{6.5}$$

$$V_{\max} = qL + F. \tag{6.6}$$

In the case of simultaneous vertical and transverse actions, the moments are calculated along the principal directions and combined vectorially:

$$M_R = \sqrt{M_y^2 + M_z^2}. \tag{6.7}$$

For a section subjected simultaneously to axial force and bending, the maximum normal stress can be estimated by:

$$\sigma_{\max} = \frac{N}{A} + \frac{M_R}{W}. \tag{6.8}$$

For a simplified normal and shear stress state, the von Mises equivalent stress is:

$$\sigma_{\text{eq}} = \sqrt{\sigma^2 + 3\tau^2}. \tag{6.9}$$

The strength verification is expressed by comparing the equivalent stress with the allowable stress. At a preliminary stage, this formulation is suitable for comparing variants; however, final sizing must include the coefficients and action combinations prescribed by the applicable standards and the actual support conditions.

7. Numerical study: influence of steady-arm length

To highlight the influence of geometry, a conservative and transparent case is considered: the concentrated force $F_d = 90$ N applied at the end of the steady arm, without including at this stage self-weight, wind and ice accretion. The assumption does not represent the final design combination, but a sensitivity test that allows the effect of length to be separated from the effect of material.

For $D = 34$ mm, $d = 26$ mm, $I = 43\,165$ mm^4 and $W = 2\,539$ mm^3, the following results:

$$M_d = F_d L. \tag{7.1}$$

$$\delta_F = \frac{F_d L^3}{3EI}. \tag{7.2}$$

The results show that, in the simplified model, the bending stress depends linearly on L, whereas the elastic displacement depends on L^3. At $L = 1600$ mm, the moment produced by the 90 N force is 144 N m, and the bending stress is approximately 56.7 MPa. The calculated displacement is approximately 13.6 mm for S235 and 40.7 mm for AlCu4Mg with $E = 70$ GPa. These values should not be interpreted as allowable displacements of the actual assembly, since the model does not include the supports, connections, joint stiffness, or the actual load distribution.

However, the displacement ratio confirms the general result: for the same geometry and loading, changing from $E = 210$ GPa to $E = 70$ GPa leads to a displacement approximately three times larger. This effect justifies section redesign if steel is to be replaced with AlCu4Mg.

L [mm]	M_d [N m]	σ_M [MPa]	δ S235 [mm]	δ AlCu4Mg (E = 70 GPa) [mm]
400	36.0	14.18	0.21	0.64
500	45.0	17.72	0.41	1.24
600	54.0	21.27	0.71	2.14
700	63.0	24.81	1.14	3.41
800	72.0	28.36	1.69	5.08
900	81.0	31.90	2.41	7.24
1000	90.0	35.44	3.31	9.93
1100	99.0	38.99	4.40	13.21
1200	108.0	42.53	5.72	17.16
1300	117.0	46.08	7.27	21.81
1400	126.0	49.62	9.08	27.24
1500	135.0	53.17	11.17	33.51
1600	144.0	56.71	13.56	40.67

8. Component integration and technological implications

Analysis of the assembly indicates that structural performance is not determined exclusively by the tubular element. The end joints, couplings, fixing plate, and clamps introduce geometric discontinuities and local stress concentrations. In particular, holes, changes in cross-section, welds, and contact areas between different materials must be included in the final numerical model.

For galvanized components, replacing steel with aluminium also requires a technological reassessment of corrosion protection and galvanic compatibility with bolts, stainless steel, CuZn15, and the other materials. In the case of the contact-wire clamp, CuZn15 should not be automatically replaced with AlCu4Mg, since the electrical and mechanical function of the component differs from that of the structural elements.

9. Structural optimization S235 → AlCu4Mg

Based on the material properties, the optimization criterion should not be formulated solely through mass minimization. A more appropriate objective function is a multi-criteria function in which mass, stress and displacement are controlled simultaneously:

$$J = w_m \frac{m}{m_0} + w_\sigma \frac{\sigma}{\sigma_0} + w_\delta \frac{\delta}{\delta_0}. \tag{9.1}$$

Where w_m, w_σ and w_δ are weights established according to the design objectives. The acceptance conditions are that the stress and displacement remain within the specified limits, while the geometry must comply with the assembly dimensions, insulation distances, access for assembly, and manufacturing requirements.

For tubular profiles, increasing the outer diameter can significantly increase the second moment of area without a proportional increase in mass.

Therefore, for AlCu4Mg, a rational strategy is to maintain the structural function and optimize the section, followed by local verification of instability and the joint areas.

In an FEA implementation, the model should contain at least: the 3D geometry of the main components, the actual support conditions, the contacts between components, the individual material properties, the relevant welds or mechanical connections, and the separate loading cases.

The FEA results must be validated by comparison with the simplified analytical case presented in this paper.

10. Results and discussion

The main result of the analysis is the clear separation between the mass advantage and the stiffness advantage. At the same volume, the density of AlCu4Mg is approximately 35.4% of that of S235, corresponding to a theoretical mass reduction of approximately 64.6%. In contrast, the elastic modulus is only 33–36% of that of steel.

For the ∅34×4 mm steady arm, the section properties are $A \approx 377$ mm^2, $I \approx 43\,165$ mm^4 and $W \approx 2\,539$ mm^3. These quantities allow a direct transition from the dimensioned geometry to stress and displacement calculations. The sensitivity to length is strong: the moment increases with L, while displacement increases with L^3.

Therefore, a mass reduction through aluminium is realistic only if the functional stiffness of the assembly is maintained. For the same loading, the approximate condition

$$E_a I_a \approx E_s I_s \tag{10.1}$$

leads to

$$I_a \approx 3 I_s \tag{10.2}$$

when $E_a = 70$ GPa is adopted. This condition provides a simple pre-dimensioning criterion.

An important limitation of the analysis is that the drawings provide all dimensions necessary to identify the components, but do not by themselves define all operating conditions and all load combinations. Therefore, the numerical values presented for the $F_d = 90$ N case are sensitivity results, not a certification of assembly safety.

11. Conclusions

- The technical documentation allows identification of a modular architecture consisting of a stay rod, brace, steady arm, fixing devices, bars, couplings, fixing plate and contact-wire clamp.
- For the direct steady arm, the drawing indicates a $\varnothing 34 \times 4$ mm tube and lengths $L = 400$–1600 mm, allowing direct calculation of section properties and length sensitivity.
- For the $\varnothing 34 \times 4$ mm tube, $A \approx 377$ mm^2, $I \approx 43\,165$ mm^4 and $W \approx 2\,539$ mm^3 are obtained; the theoretical mass of the S235 tube is ≈ 2.96 kg/m, consistent with the values in the drawing table.
- For the same geometry, AlCu4Mg can theoretically reduce mass by approximately 64.6%, but the lower elastic modulus leads to displacements approximately three times larger for $E = 70$ GPa.
- For the conservative case $F_d = 90$ N applied at the end, the moment at $L = 1600$ mm is 144 N m, and the bending stress for the analyzed section is ≈ 56.7 MPa.
- Replacement of S235 with AlCu4Mg must be performed selectively and accompanied by optimization of geometry, joints and corrosion protection/galvanic compatibility.
- The recommended next step is to build a complete FEA model and validate it against a simplified analytical model, followed by multi-criteria mass–strength–stiffness optimization.

12. Study limitations and future directions

The study is analytical and comparative in nature. Future directions include: parametric 3D modeling; static and modal FEA analysis; verification of stress concentrations around holes and joints; fatigue verification; assessment of the local stability of thin-walled aluminium profiles; analysis of thermal and galvanic effects; topological or cross-sectional optimization while preserving the functional dimensions.

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