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Decomposition of Continuity in a Fuzzy Sequential Topological Space

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Abstract

A new class of fuzzy sequential sets called fs-preopen sets is introduced and characterized. An fs-precontinuous mapping and strongly fs-precontinuous mapping are defined and studied. A necessary and sufficient condition under which an fs-preopen set is fs-open, has been established. Apart from that, different kinds of sets, namely, fs α-open set, fs δ-set, fs $\mathcal{A}$-set, locally fs-closed set, fs S-preopen set have been studied. The interrelationships between them are investigated. Using them and the associated continuities, various decompositions of fs-continuity have been established.

Keywords: Fuzzy sequential topological spaces, fs-preopen, fs α-open, fs δ-set, fs $\mathcal{A}$-set, locally fs-closed, fs S-preopen sets.
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1. Introduction

In the last few decades, there has been interests in the study of generalized open sets and generalized continuity in topological spaces. Various authors studied different kinds of generalized open sets in topological spaces. In the fuzzy setting, fuzzy semi-open sets and fuzzy semicontinuity were introduced and studied by K. K. Azad (Azad, 1981), fuzzy pre-open sets (Mashhour *et al.*, 1982) by A. S. Mashhour, M. E. Abd El-Monsef and S. N. EI-Deeb. Again, fs-semiopen sets and fs-semicontinuity have been studied in (Tamang & Sarkar, 2016).

The purpose of this work is to study different generalized open sets and the associated continuous functions in a fuzzy sequential topological space.

Apart from the introduction in Section 1, Section 2 is devoted to the study of preopen sets and precontinuity in a fuzzy sequential topological space. An important result from this Section is a

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necessary and sufficient condition for a preopen set to be open. Section 3 deals with the introduction and study of some more generalized open sets and their respective continuities. Finally the Section has been concluded with some decompositions of continuity.

Throughout the paper X will denote a non empty set and I the unit interval $[0, 1]$. Sequences of fuzzy sets in X called fuzzy sequential sets (fs-sets) will be denoted by the symbols $A_f(s)$, $B_f(s)$, $C_f(s)$, etc. An fs-set $X_f^l(s)$ is a sequence of fuzzy sets $\{X_f^n\}_n$ where $l \in I$ and $X_f^n(x) = l$, for all $x \in X$, $n \in \mathbb{N}$,

A family $\delta(s)$ of fuzzy sequential sets on a set X satisfying the properties

(i) $X_f^r(s) \in \delta(s)$ for $r = 0$ and 1,

(ii) $A_f(s), B_f(s) \in \delta(s) \Rightarrow A_f(s) \wedge B_f(s) \in \delta(s)$ and

(iii) for any family $\{A_{fj}(s) \in \delta(s),\ j \in J\}$, $\bigvee_{j \in J} A_{fj}(s) \in \delta(s)$

is called a fuzzy sequential topology (FST) on X and the ordered pair $(X, \delta(s))$ is called fuzzy sequential topological space (FSTS) (Singha *et al.*, 2014). The members of $\delta(s)$ are called open fuzzy sequential (fs-open) sets in X. Complement of an open fuzzy sequential set in X is called closed fuzzy sequential (fs-closed) set in X. In an FSTS, the closure, interior, continuous functions, semiopen sets etc. are defined in the usual manner (See (Singha *et al.*, 2014), (Tamang & Sarkar, 2016), (Tamang *et al.*, 2016)).

2. FS-preopen sets and FS-precontinuity

Definition 2.1. (i) An fs-set $A_f(s)$ in an FSTS, is said to be an fs-preopen set if $A_f(s) \leq (\overline{A_f(s)})^o$.
(ii) An fs-set $A_f(s)$ in an FSTS, is said to be an fs-preclosed set if its complement is fs-preopen or equivalently if $\overline{\overset{o}{A}_f(s)} \leq A_f(s)$.

If $A_f(s)$ is both fs-preopen and fs-preclosed, then it is called an fs-preclopen set.

Definition 2.2. An fs-set $A_f(s)$ is called fs-dense in an FSTS $(X, \delta(s))$, if $\overline{A_f(s)} = X_f^1(s)$.

Fundamental properties of fs-preopen (fs-preclosed) sets are:

- Every fs-open (fs-closed) set is fs-preopen (fs-preclosed).
- Arbitrary union (intersection) of fs-preopen (fs-preclosed) sets is fs-preopen (fs-preclosed).

Example 2.1 shows that an fs-preopen (fs-preclosed) set may not be fs-open (fs-closed), the intersection (union) of any two fs-preopen (fs-preclosed) sets need not be an fs-preopen (fs-preclosed) set. Unlike in a general topological space, the intersection of an fs-preopen set with an fs-open set may fail to be an fs-preopen set.

Example 2.1. Consider the fs-sets $A_f(s)$, $B_f(s)$ and $C_f(s)$ in $X = [0, 1]$, defined as follows:

$$\begin{aligned} A_f^1(x) &= 0, \text{ if } 0 \leq x \leq \frac{1}{2} \\ &= \frac{1}{4}, \text{ if } \frac{1}{2} < x \leq 1 \\ \text{and } A_f^n &= \overline{1} \text{ for all } n \neq 1. \end{aligned}$$

$$\begin{aligned} B^1_f(x) &= \frac{1}{2}, \text{ if } 0 \le x \le \frac{1}{2} \\ &= 0, \text{ if } \frac{1}{2} < x \le 1 \\ \text{and } B^n_f &= \overline{0} \text{ for all } n \ne 1. \end{aligned}$$

$$\begin{aligned} C^1_f(x) &= \frac{3}{4}, \text{ if } 0 \le x \le \frac{1}{2} \\ &= 1, \text{ if } \frac{1}{2} < x \le 1 \\ \text{and } C^n_f &= \overline{0} \text{ for all } n \ne 1. \end{aligned}$$

Let $\delta(s) = \{A_f(s), B_f(s), A_f(s) \vee B_f(s), X^0_f(s), X^1_f(s)\}$. Then $(X, \delta(s))$ is an FSTS. Now,
(i) $A_f(s)$ and $C_f(s)$ are fs-preopen sets but their intersection is not fs-preopen.
(ii) $C_f(s)$ is fs-preopen but is not fs-open.

Theorem 2.1. *Let $(X, \delta(s))$ be an FSTS. An fs-set $A_f(s)$ is fs-preopen if and only if there exists an fs-open set $O_f(s)$ in X such that $A_f(s) \le O_f(s) \le \overline{A_f(s)}$.*

Proof. Straightforward. □

Corollary 2.1. *Let $(X, \delta(s))$ be an FSTS. An fs-set $A_f(s)$ is fs-preclosed if and only if there exists an fs-closed set $C_f(s)$ in X such that $\overset{o}{A}_f(s) \le C_f(s) \le A_f(s)$.*

Proof. Straightforward. □

Theorem 2.2. *An fs-set is fs-clopen (both fs-closed and fs-open) if and only if it is fs-closed and fs-preopen.*

Proof. Proof is omitted. □

Theorem 2.3. *In an FSTS, every fs-set is fs-preopen if and only if every fs-open set is fs-closed.*

Proof. Suppose every fs-set in an FSTS $(X, \delta(s))$, is fs-preopen and let $A_f(s)$ be an fs-open set. Then, $A^c_f(s) = \overline{A^c_f(s)}$ is fs-preopen and hence $\overline{A^c_f(s)} \le (\overline{\overline{A^c_f(s)}})^o = (\overline{A^c_f(s)})^o = (A^c_f(s))^o$. Thus, $A^c_f(s)$ is fs-open and hence $A_f(s)$ is fs-closed.

Conversely, suppose every fs-open set is fs-closed and let $A_f(s)$ be any fs-set. By the assumption, $\overline{A_f(s)} = (\overline{A_f(s)})^o$ and hence $A_f(s)$ is fs-preopen. □

Theorem 2.4. (*a*) *Closure of an fs-preopen set is fs-regular closed.*
(*b*) *Interior of an fs-preclosed set is fs-regular open.*

Proof. We prove only (a). Let $A_f(s)$ be an fs-preopen set in X. Since $(\overline{A_f(s)})^o \le \overline{A_f(s)}$, we have $\overline{(\overline{A_f(s)})^o} \le \overline{\overline{A_f(s)}} = \overline{A_f(s)}$. Now $A_f(s)$ being fs-preopen, $A_f(s) \le (\overline{A_f(s)})^o$ and hence $\overline{A_f(s)} \le \overline{(\overline{A_f(s)})^o}$. Thus, $\overline{A_f(s)}$ is fs-regular closed. □

The set of all fs-preopen sets in X, is denoted by $FSPO(X)$.

Theorem 2.5. *In an FSTS* $(X, \delta(s))$, (i) $\delta(s) \subseteq FSPO(X)$, (ii) *If* $V_f(s) \in FSPO(X)$ *and* $U_f(s) \leq V_f(s) \leq \overline{U_f(s)}$, *then* $U_f(s) \in FSPO(X)$.

Proof. (i) Follows from definition.

(ii) Let $V_f(s) \in FSPO(X)$, that is, $V_f(s) \leq (\overline{V_f(s)})^o$. We have,

$$U_f(s) \leq V_f(s) \leq \overline{U_f(s)}$$

Therefore, $U_f(s) \leq V_f(s) \leq (\overline{V_f(s)})^o \leq (\overline{U_f(s)})^o$. Hence the result. □

Definition 2.3. An fs-set $A_f(s)$ in an FSTS, is called an fs-preneighbourhood of an fs-point $P_f(s) = (p^M_{fx}, r)$, if there exists an fs-preopen set $B_f(s)$ such that $P_f(s) \leq B_f(s) \leq A_f(s)$.

Theorem 2.6. *For an fs-set* $A_f(s)$ *in an FSTS* $(X, \delta(s))$, *the following are equivalent:*
(i) $A_f(s)$ *is fs-preopen.*
(ii) *There exists an fs-regular open set* $B_f(s)$ *containing* $A_f(s)$ *such that* $\overline{A_f(s)} = \overline{B_f(s)}$.
(iii) ${}_scl(A_f(s)) = (\overline{A_f(s)})^o$.
(iv) *The semi-closure of* $A_f(s)$ *is fs-regular open.*
(v) $A_f(s)$ *is an fs-preneighbourhood of each of its fs-points.*

Proof. $(i) \Rightarrow (ii)$ Let $A_f(s)$ be fs-preopen. This implies

$$\begin{aligned} & A_f(s) \leq (\overline{A_f(s)})^o \leq \overline{A_f(s)} \\ \Rightarrow\ & \overline{A_f(s)} \leq \overline{(\overline{A_f(s)})^o} \leq \overline{A_f(s)} \\ \Rightarrow\ & \overline{(\overline{A_f(s)})^o} = \overline{A_f(s)} \\ \Rightarrow\ & \overline{A_f(s)} = \overline{B_f(s)} \end{aligned}$$

where $B_f(s) = (\overline{A_f(s)})^o$ is an fs-regular open set containing $A_f(s)$.

$(ii) \Rightarrow (iii)$ Let $\overline{A_f(s)} = \overline{B_f(s)}$, where $B_f(s)$ is an fs-regular open set containing $A_f(s)$. Then,

$$A_f(s) \leq B_f(s) = (\overline{B_f(s)})^o = (\overline{A_f(s)})^o$$

Also, $(\overline{A_f(s)})^o$ is fs-semiclosed. Let $C_f(s)$ be an fs-semiclosed set containing $A_f(s)$. Thus,

$$(\overline{A_f(s)})^o \leq (\overline{C_f(s)})^o \leq C_f(s).$$

Hence ${}_scl(A_f(s)) = (\overline{A_f(s)})^o$.

$(iii) \Rightarrow (iv)$ Obvious.

$(iv) \Rightarrow (i)$ Suppose ${}_scl(A_f(s))$ is fs-regular open. Now,

$$\begin{aligned} & A_f(s) \leq {}_scl(A_f(s)) \\ \Rightarrow\ & (\overline{A_f(s)})^o \leq (\overline{{}_scl(A_f(s))})^o = {}_scl(A_f(s)) \leq (\overline{A_f(s)})^o \\ \Rightarrow\ & A_f(s) \leq {}_scl(A_f(s)) = (\overline{A_f(s)})^o \end{aligned}$$

Thus, $A_f(s)$ is fs-preopen.

$(i) \Rightarrow (v)$ and $(v) \Rightarrow (i)$ are obvious. □

Corollary 2.2. *An fs-set is fs-regular open if and only if it is fs-semiclosed and fs-preopen.*

Proof. Proof is omitted. □

Theorem 2.7. *In an FSTS $(X, \delta(s))$, the following are equivalent:*
(i) $\overline{A_f(s)} \in \delta(s)$ *for all* $A_f(s) \in \delta(s)$.
(ii) *Every fs-regular closed set in X is fs-preopen.*
(iii) *Every fs-semiopen set in X is fs-preopen.*
(iv) *The closure of every fs-preopen set in X is fs-open.*
(v) *The closure of every fs-preopen set in X is fs-preopen.*

Proof. $(i) \Rightarrow (ii)$ Let $A_f(s)$ be an fs-regular closed set, that is, $\overline{\overset{o}{A}_f(s)} = A_f(s)$. By (i), $A_f(s) \in \delta(s)$ and hence $A_f(s)$ is fs-preopen.

$(ii) \Rightarrow (iii)$ Let $A_f(s)$ be an fs-semiopen set, that is, $A_f(s) \leq \overline{\overset{o}{A}_f(s)}$. By (ii), $\overline{\overset{o}{A}_f(s)}$ is fs-preopen. Also, we have, $A_f(s) \leq \overline{\overset{o}{A}_f(s)} \leq \overline{A_f(s)}$. Thus, $A_f(s)$ is fs-preopen.

$(iii) \Rightarrow (iv)$ Let $A_f(s)$ be an fs-preopen set, that is, $A_f(s) \leq (\overline{A_f(s)})^o$. This implies, $\overline{A_f(s)} \leq \overline{(\overline{A_f(s)})^o}$. Thus, $\overline{A_f(s)}$ being fs-semiopen, is fs-preopen and the result follows.

$(iv) \Rightarrow (v)$ Obvious.

$(v) \Rightarrow (i)$ Let $A_f(s) \in \delta(s)$. Then, $A_f(s)$ is fs-preopen and hence $\overline{A_f(s)}$ is fs-preopen. Therefore, $\overline{A_f(s)} \leq (\overline{A_f(s)})^o \leq \overline{A_f(s)}$ and hence $\overline{A_f(s)} \in \delta(s)$. □

Theorem 2.8. *In an FSTS $(X, \delta(s))$, the following are equivalent:*
(i) *Every non-zero fs-open set is fs-dense.*
(ii) *For every non-zero fs-preopen set $A_f(s)$, we have $_scl(A_f(s)) = X^1_f(s)$.*
(iii) *Every non-zero fs-preopen set is fs-dense.*

Proof. $(i) \Rightarrow (ii)$ Let $A_f(s)$ be a non-zero fs-preopen set. By Theorem 2.6 (iii), $_scl(A_f(s)) = (\overline{A_f(s)})^o$. Also, there exists an fs-open set $O_f(s)$ such that $A_f(s) \leq O_f(s) \leq \overline{A_f(s)}$. By (i), $\overline{O_f(s)} = X^1_f(s)$. Therefore, $\overline{A_f(s)} = X^1_f(s)$ and hence $_scl(A_f(s)) = X^1_f(s)$.

$(ii) \Rightarrow (iii)$ Easy to prove.

$(iii) \Rightarrow (i)$ Since every fs-open set is fs-preopen, the proof is straightforward. □

Definition 2.4. Let $A_f(s)$ be an fs-set in an FSTS $(X, \delta(s))$. We define fs-preclosure $_pcl(A_f(s))$ and fs-preinterior $_pint(A_f(s))$ of $A_f(s)$ by

$$_pcl(A_f(s)) = \wedge\{B_f(s); A_f(s) \leq B_f(s) \text{ and } B^c_f(s) \in FSPO(X)\}$$
$$_pint(A_f(s)) = \vee\{C_f(s); C_f(s) \leq A_f(s) \text{ and } C_f(s) \in FSPO(X)\}$$

Clearly, $_pcl(A_f(s))$ is the smallest fs-preclosed set containing $A_f(s)$ and $_pint(A_f(s))$ is the largest fs-preopen set contained in $A_f(s)$. Further,
(i) $A_f(s) \leq {_pcl}(A_f(s)) \leq \overline{A_f(s)}$ and $\overset{o}{A}_f(s) \leq {_pint}(A_f(s)) \leq A_f(s)$.
(ii) $A_f(s)$ is fs-preopen if and only if $A_f(s) = {_pint}(A_f(s))$
(iii) $A_f(s)$ is fs-preclosed if and only if $A_f(s) = {_pcl}(A_f(s))$
(iv) $A_f(s) \leq B_f(s) \Rightarrow {_pint}(A_f(s)) \leq {_pint}(B_f(s))$ and $_pcl(A_f(s)) \leq {_pcl}(B_f(s))$.

Definition 2.5. A mapping $g : (X, \delta(s)) \to (Y, \delta'(s))$ is said to be
(i) fs-precontinuous if $g^{-1}(B_f(s))$ is fs-preopen in X, for every $B_f(s) \in \delta'(s)$.
(ii) fs-preopen if $g(A_f(s))$ is fs-preopen in Y, for every $A_f(s) \in \delta(s)$.
(iii) fs-preclosed if $g(A_f(s))$ is fs-preclosed in Y, for every fs-closed set $A_f(s)$ in X.

It is easy to check that an fs-continuous (fs-open, fs-closed) function is fs-precontinuous (fs-preopen, fs-preclosed). That the converse may not be true, is shown by Example 2.2.

Example 2.2. Consider the fs-sets $A_f(s)$, $B_f(s)$, $C_f(s)$ in a set X, defined as follows:

$$\begin{aligned} A_f(s) &= \left\{\overline{\frac{1}{4}}, \overline{1}, \overline{1}, ...\right\} \\ B_f(s) &= \left\{\overline{\frac{1}{2}}, \overline{0}, \overline{0}, ...\right\} \\ C_f(s) &= \left\{\overline{\frac{3}{8}}, \overline{1}, \overline{1}, ...\right\} \end{aligned}$$

Let $\delta(s) = \{A_f(s), B_f(s), A_f(s) \wedge B_f(s), A_f(s) \vee B_f(s), X_f^0(s), X_f^1(s)\}$. Then $(X, \delta(s))$ is an FSTS. Let $\delta'(s) = \{C_f(s), X_f^0(s), X_f^1(s)\}$ and define $g : (X, \delta(s)) \to (X, \delta'(s))$ by $g(x) = x$ for all $x \in X$. The function g is fs-precontinuous but not fs-continuous.

Again, the map $h : (X, \delta'(s)) \to (X, \delta(s))$ defined by $h(x) = x$ for all $x \in X$, is both fs-preopen and fs-preclosed but neither fs-open nor fs-closed.

Theorem 2.9. *Let $g : (X, \delta(s)) \to (Y, \eta(s))$ be a map. Then the following conditions are equivalent:*
(i) *g is fs-precontinuous.*
(ii) *the inverse image of an fs-closed set in Y under g, is fs-preclosed in X.*
(iii) *For any fs-set $A_f(s)$ in X, $g({}_pcl(A_f(s))) \leq \overline{g(A_f(s))}$.*

Proof. (i) $\Rightarrow$ (ii) Suppose g be an fs-precontinuous map and $B_f(s)$ be an fs-closed set in Y. Then,

$$\begin{aligned} & B_f^c(s) \text{ is fs-open in } Y \\ \Rightarrow\ & (g^{-1}(B_f(s)))^c = g^{-1}(B_f^c(s)) \text{ is fs-preopen in } X \\ \Rightarrow\ & g^{-1}(B_f(s)) \text{ is fs-preclosed in } X. \end{aligned}$$

(ii) $\Rightarrow$ (iii) Let $A_f(s)$ be an fs-set in X. Then, $g^{-1}(\overline{g(A_f(s))})$ is fs-preclosed in X and hence $g^{-1}(\overline{g(A_f(s))}) = {}_pcl(g^{-1}(\overline{g(A_f(s))}))$. Again,

$$\begin{aligned} & A_f(s) \leq g^{-1}(g(A_f(s))) \\ \Rightarrow\ & {}_pcl(A_f(s)) \leq {}_pcl(g^{-1}(g(A_f(s)))) \leq g^{-1}(\overline{g(A_f(s))}) \\ \Rightarrow\ & g({}_pcl(A_f(s))) \leq g(g^{-1}(\overline{g(A_f(s))})) \leq \overline{g(A_f(s))}. \end{aligned}$$

(iii) $\Rightarrow$ (i) Let $B_f(s)$ be an fs-open set in Y. Then for the fs-closed set $B_f^c(s)$, we have

$$g({}_pcl(g^{-1}(B_f^c(s)))) \leq \overline{g(g^{-1}(B_f^c(s)))} \leq \overline{B_f^c(s)} = B_f^c(s)$$

Thus, ${}_pcl(g^{-1}(B^c_f(s))) \leq g^{-1}(B^c_f(s))$. Therefore, ${}_pcl(g^{-1}(B^c_f(s))) = g^{-1}(B^c_f(s))$ and hence $(g^{-1}(B_f(s)))^c = g^{-1}(B^c_f(s))$ is fs-preclosed in X. □

In Theorem 2.8 (Tamang & Sarkar, 2016), it has been proved that the inverse image of an fs-semiopen set is fs-semiopen, under an fs-semicontinuous open map. The next Theorem shows that the result is true even if we take an fs-semicontinuous preopen map.

Theorem 2.10. *Suppose $g : (X, \delta(s)) \to (Y, \eta(s))$ be an fs-semicontinuous preopen mapping. Then the inverse image of every fs-semiopen set in Y under g, is fs-semiopen in X.*

Proof. Let $B_f(s)$ be an fs-semiopen set in Y. Then there exists an fs-open set $O_f(s)$ in Y such that

$$\begin{aligned} & O_f(s) \leq B_f(s) \leq \overline{O_f(s)} \\ \Rightarrow \quad & g^{-1}(O_f(s)) \leq g^{-1}(B_f(s)) \leq g^{-1}(\overline{O_f(s)}) \end{aligned}$$

We claim that $g^{-1}(\overline{O_f(s)}) \leq \overline{g^{-1}(O_f(s))}$. Let $P_f(s) \in g^{-1}(\overline{O_f(s)})$. This implies $g(P_f(s)) \in \overline{O_f(s)}$. Consider a weak open Q-nbd $U_f(s)$ of $P_f(s)$, then $\overline{g(U_f(s))}$ is a weak Q-nbd of $g(P_f(s))$. Therefore,

$$\begin{aligned} & \overline{g(U_f(s))} q_w O_f(s) \\ \Rightarrow \quad & W_f(s) q_w O_f(s) \text{ where } W_f(s) = \overline{g(U_f(s))} \\ \Rightarrow \quad & W^n_f(y) + O^n_f(y) > 1 \text{ for some } y \in Y \\ \Rightarrow \quad & O_f(s) \text{ is a weak open Q-nbd of the fs-point } (p^n_{fy}, W^n_f(y)) \\ \Rightarrow \quad & g(U_f(s)) q_w O_f(s) \\ \Rightarrow \quad & U_f(s) q_w g^{-1}(O_f(s)) \\ \Rightarrow \quad & P_f(s) \in \overline{g^{-1}(O_f(s))}. \end{aligned}$$

Thus $g^{-1}(O_f(s)) \leq g^{-1}(B_f(s)) \leq \overline{g^{-1}(O_f(s))}$. Since $g^{-1}(O_f(s))$ is fs-semiopen, $g^{-1}(B_f(s))$ is fs-semiopen. □

Corollary 2.3. *Suppose $g : (X, \delta(s)) \to (Y, \eta(s))$ be an fs-semicontinuous preopen mapping. Then the inverse image of every fs-semiclosed set in Y under g, is fs-semiclosed in X.*

Proof. The proof is omitted. □

Corollary 2.4. *Suppose $g : (X, \delta(s)) \to (Y, \eta(s))$ be an fs-semicontinuous preopen map and $h : (Y, \eta(s)) \to (Z, \delta'(s))$ be fs-semicontinuous. Then hog is fs-semicontinuous.*

Proof. The proof is omitted. □

Theorem 2.11. *Suppose $g : (X, \delta(s)) \to (Y, \eta(s))$ be an fs-precontinuous preopen mapping. Then the inverse image of every fs-preopen set in Y under g, is fs-preopen in X.*

Proof. Let $B_f(s)$ be an fs-preopen set in Y. Then there exists an fs-open set $O_f(s)$ in Y such that

$$\begin{aligned} & B_f(s) \leq O_f(s) \leq \overline{B_f(s)} \\ \Rightarrow \quad & g^{-1}(B_f(s)) \leq g^{-1}(O_f(s)) \leq g^{-1}(\overline{B_f(s)}). \end{aligned}$$

As in Theorem 2.10, we have $g^{-1}(\overline{B_f(s)}) \leq \overline{g^{-1}(B_f(s))}$. Thus $g^{-1}(B_f(s)) \leq g^{-1}(O_f(s)) \leq \overline{g^{-1}(B_f(s))}$, where $g^{-1}(O_f(s))$ is fs-preopen. Hence $g^{-1}(B_f(s))$ is fs-preopen. □

Corollary 2.5. *Suppose $g : (X, \delta(s)) \to (Y, \eta(s))$ be an fs-precontinuous preopen mapping. Then the inverse image of every fs-preclosed set in Y under g, is fs-preclosed in X.*

Proof. The proof is omitted. □

Corollary 2.6. *Suppose $g : (X, \delta(s)) \to (Y, \eta(s))$ be an fs-precontinuous preopen map and $h : (Y, \eta(s)) \to (Z, \delta'(s))$ be an fs-precontinuous map. Then hog is fs-precontinuous.*

Proof. The proof is omitted. □

Theorem 2.12. *Suppose $g : (X, \delta(s)) \to (Y, \eta(s))$ be an fs-continuous open map. Then the g-image of an fs-preopen set in X, is fs-preopen in Y.*

Proof. Let $A_f(s)$ be an fs-preopen set in X. Then there exists an fs-open set $O_f(s)$ in X such that $A_f(s) \leq O_f(s) \leq \overline{A_f(s)}$. This implies $g(A_f(s)) \leq g(O_f(s)) \leq \overline{g(A_f(s))}$. Since $g(O_f(s))$ is fs-open in Y, $g(A_f(s))$ is fs-preopen. □

Corollary 2.7. *Pre-openness in an FSTS, is a topological property.*

Proof. Proof follows from Theorem 2.12. □

Theorem 2.13. *Let $g : (X, \delta(s)) \to (Y, \eta(s))$ and $h : (Y, \eta(s)) \to (Z, \delta'(s))$ be two mappings, such that hog is fs-preclosed. Then g is fs-preclosed if h is an injective fs-precontinuous preopen mapping.*

Proof. Let $A_f(s)$ be an fs-closed set in X. Then, $hog(A_f(s))$ is fs-preclosed in Z and hence $g(A_f(s)) = h^{-1}(hog(A_f(s)))$ is fs-preclosed in Y. □

Theorem 2.14. *If $g : (X, \delta(s)) \to (Y, \delta'(s))$ be fs-precontinuous and $h : (Y, \delta'(s)) \to (Z, \eta(s))$ be fs-continuous, then hog $: (X, \delta(s)) \to (Z, \eta(s))$ is fs-precontinuous.*

Proof. Omitted. □

Previously, we showed that the intersection of any two fs-preopen sets may not be fs-preopen and an fs-preopen set may not be fs-open. Now, we investigate and establish conditions, under which the intersection of any two fs-preopen sets is fs-preopen and conditions, under which an fs-preopen set is fs-open.

Theorem 2.15. *The intersection of any two fs-preopen sets is fs-preopen if the closure is preserved under finite intersection.*

Proof. Proof is simple and hence omitted. □

Theorem 2.16. *In an FSTS $(X, \delta(s))$, if every fs-set is either fs-open or fs-closed, then every fs-preopen set in X is fs-open.*

Proof. Let $A_f(s)$ be an fs-preopen set in X. If $A_f(s)$ is not fs-open, then it is fs-closed and hence $\overline{A_f(s)} = A_f(s)$. Therefore, $A_f(s) \leq (\overline{A_f(s)})^o = \overset{o}{A}_f(s)$ and hence the theorem. □

For a fuzzy sequential topological space $(X, \delta(s))$, $\delta^*(s)$ will denote the fuzzy sequential topology on X, obtained by taking $FSPO(X)$ as a subbase.

Definition 2.6. A mapping $g : (X, \delta(s)) \to (Y, \eta(s))$ is called strongly fs-precontinuous if the inverse image of each fs-preopen set in Y is fs-open in X.

By the definition of a strong fs-precontinuous mapping, the following two results are obvious.

Theorem 2.17. (i) *A map $g : (X, \delta(s)) \to (Y, \eta(s))$ is strongly fs-precontinuous if and only if $g : (X, \delta(s)) \to (Y, \eta^*(s))$ is fs-continuous.*
(ii) *If $g : (X, \delta(s)) \to (Y, \eta(s))$ is strongly fs-precontinuous, then it is fs-continuous.*

Remark. Converse of (ii) of Theorem 2.17 may not be true, as is shown by the following Example.

Example 2.3. Consider the fs-sets $A_f(s)$, $B_f(s)$, $C_f(s)$ in a set X, defined as follows:

$$A_f(s) = \left\{\overline{\frac{1}{4}}, \bar{1}, \bar{1}, ...\right\}$$

$$B_f(s) = \left\{\overline{\frac{1}{2}}, \bar{0}, \bar{0}, ...\right\}$$

$$C_f(s) = \left\{\overline{\frac{3}{8}}, \bar{1}, \bar{1}, ...\right\}$$

Let $\delta(s) = \{A_f(s), B_f(s), A_f(s) \wedge B_f(s), A_f(s) \vee B_f(s), X_f^0(s), X_f^1(s)\}$. Then $(X, \delta(s))$ is an FSTS. Consider the identity map $id : (X, \delta(s)) \to (X, \delta(s))$. Then, id is fs-continuous but not strongly fs-precontinuous, as the inverse image of fs-preopen set $C_f(s)$ is not fs-open.

We conclude the section with a necessary and sufficient condition for an fs-preopen set to be fs-open.

Theorem 2.18. *In an FSTS $(Y, \eta(s))$, the following are equivalent:*
(i) *Every fs-preopen set in Y is fs-open.*
(ii) *Every fs-continuous function $g : (X, \delta(s)) \to (Y, \eta(s))$ is strongly fs-precontinuous, where $(X, \delta(s))$ is any FSTS.*

Proof. $(i) \Rightarrow (ii)$ is straightforward.
$(ii) \Rightarrow (i)$ The identity map $g : (Y, \eta(s)) \to (Y, \eta(s))$ is fs-continuous and hence is strongly fs-precontinuous. Let $B_f(s)$ be an fs-preopen set in Y, then $B_f(s) = g^{-1}(B_f(s))$ is fs-open in Y. □

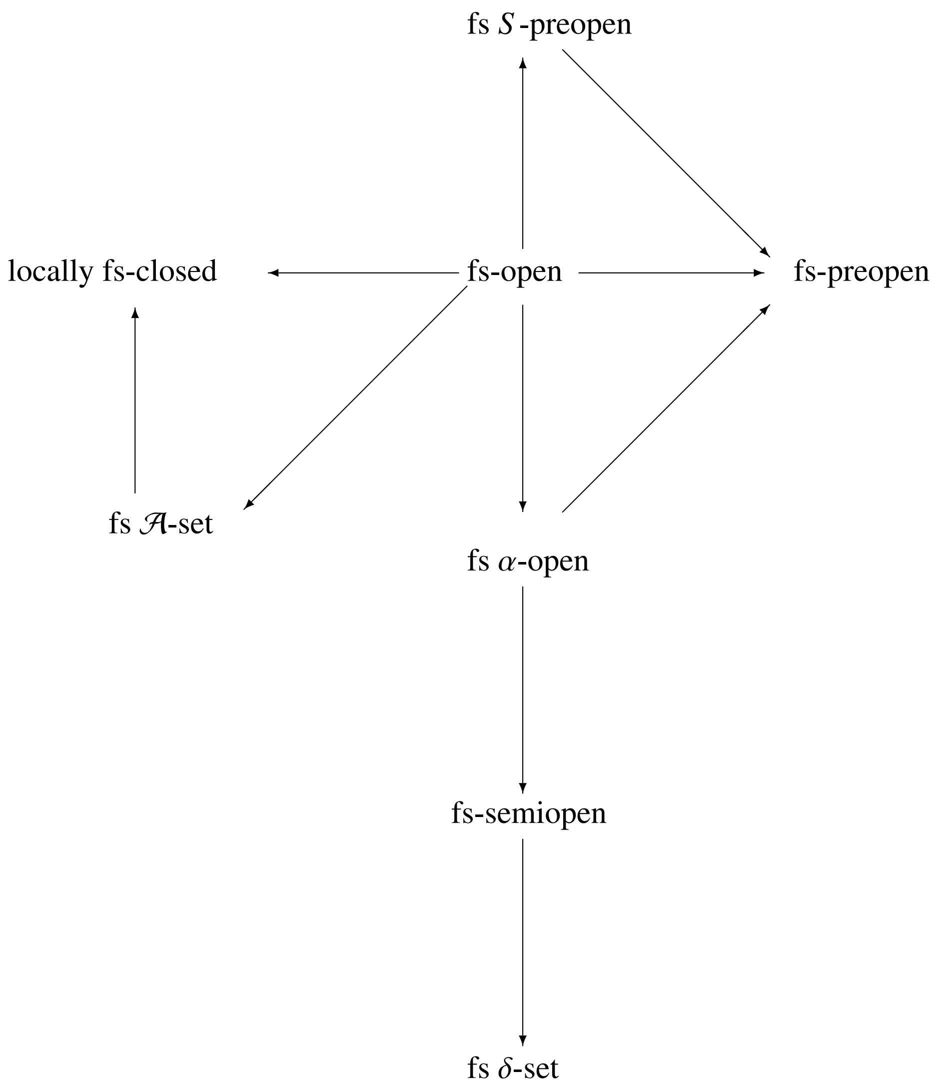
3. Decomposition of Continuity

In (Tamang & Sarkar, 2016) and in the last section, nearly open sets like fs-semiopen, fs-preopen and fs-regular open sets in a fuzzy sequential topological space have been studied. Here, we study some more of such sets and the respective continuities and the section is concluded establishing a few decompositions of fs-continuity. In this section, for our convenience, we denote the closure and interior by *cl* and *int* respectively.

Definition 3.1. Let $(X, \delta(s))$ be an FSTS. An fs-set $A_f(s)$ is called
(i) fs α-open if $A_f(s) \leq int\ cl\ int A_f(s)$;
(ii) locally fs-closed if $A_f(s) = U_f(s) \wedge V_f(s)$, where $U_f(s)$ is fs-open and $V_f(s)$ is fs-closed;
(iii) an fs $\mathcal{A}$-set if $A_f(s) = U_f(s) \wedge V_f(s)$, where $U_f(s)$ is fs-open and $V_f(s)$ is fs-regular closed;
(iv) an fs δ-set if $int\ cl A_f(s) \leq cl\ int A_f(s)$;
(v) fs S-preopen if $A_f(s)$ is fs-preopen and $A_f(s) = U_f(s) \wedge V_f(s)$, where $U_f(s)$ is fs-open and $int V_f(s)$ is fs-regular open.

We denote the collection of all fs α-open sets, fs-semiopen sets, fs-preopen sets, fs $\mathcal{A}$-sets, fs S-preopen sets, locally fs-closed sets and fs δ-sets in an FSTS $(X, \delta(s))$, by $\alpha(X)$, $FSSO(X)$, $FSPO(X)$, $\mathcal{A}(X)$, $FSSPO(X)$, $FSLC(X)$ and $\delta(X)$ respectively.

The relationships among different fs-sets defined above, are given by the following diagram:



The implications in the above diagram are not reversible. To show this, here we give examples. In (Tamang & Sarkar, 2016) and in Section 2 respectively, it is already shown that an fs-semiopen and an fs-preopen set may not be fs-open.

Example 3.1. Example to show that an fs α-open set may not be fs-open.

Consider the fs-sets $A_f(s)$, $B_f(s)$, $C_f(s)$ in a set X, defined as follows:

$$A_f(s) = \left\{\overline{\frac{1}{4}}, \overline{1}, \overline{1}, ...\right\}$$
$$B_f(s) = \left\{\overline{\frac{1}{2}}, \overline{0}, \overline{0}, ...\right\}$$
$$C_f(s) = \left\{\overline{\frac{3}{8}}, \overline{1}, \overline{1}, ...\right\}$$

Let $\delta(s) = \{A_f(s), B_f(s), A_f(s) \wedge B_f(s), A_f(s) \vee B_f(s), X_f^0(s), X_f^1(s)\}$. Then $(X, \delta(s))$ is an FSTS, where $C_f(s)$ is fs α-open but is not fs-open.

Example 3.2. Example to show that a locally fs-closed set may not be fs-open.

Consider the fs-sets $A_f(s)$, $B_f(s)$ in a set X, defined as follows:

$$A_f(s) = \left\{\overline{\frac{1}{4}}, \overline{\frac{3}{4}}, \overline{\frac{1}{4}}, \overline{\frac{3}{4}}, ...\right\}$$
$$B_f(s) = \left\{\overline{\frac{1}{4}}, \overline{\frac{1}{4}}, \overline{\frac{1}{4}}, \overline{\frac{1}{4}}, ...\right\}$$

Let $\delta(s) = \{A_f(s), X_f^0(s), X_f^1(s)\}$. Then $(X, \delta(s))$ is an FSTS, where $B_f(s)$ is locally fs-closed but not fs-open.

Example 3.3. Example to show that an fs $\mathcal{A}$-set may not be fs-open.

Consider the fs-sets $A_f(s)$, $B_f(s)$, $C_f(s)$, $D_f(s)$ in a set X, defined as follows:

$$A_f(s) = \left\{\overline{1}, \overline{\frac{1}{2}}, \overline{1}, \overline{\frac{1}{2}}, ...\right\}$$
$$B_f(s) = \left\{\overline{\frac{1}{2}}, \overline{0}, \overline{\frac{1}{2}}\overline{0}, ...\right\}$$
$$C_f(s) = \left\{\overline{\frac{1}{2}}, \overline{\frac{1}{2}}, \overline{\frac{1}{2}}, ...\right\}$$
$$D_f(s) = \left\{\overline{\frac{1}{2}}, \overline{1}, \overline{\frac{1}{2}}, \overline{1}, ...\right\}$$

Consider the fuzzy sequential topological space $(X, \delta(s))$, where $\delta(s) = \{A_f(s), B_f(s), X_f^0(s), X_f^1(s)\}$. Here, $C_f(s) = A_f(s) \wedge D_f(s)$, where $A_f(s)$ is fs-open and $D_f(s)$ is fs-regular closed. Hence $C_f(s)$ is an fs $\mathcal{A}$-set but is not fs-open.

Example 3.4. Example to show that a locally fs-closed set may not be an fs $\mathcal{A}$-set.
Consider the FSTS $(X, \delta(s))$, given in Example 3.2. Here, $B_f(s)$ is a locally fs-closed set but not an fs $\mathcal{A}$-set.

Example 3.5. Example to show that an fs-semiopen set may not be an fs $\mathcal{A}$-set.
Consider the FSTS $(X, \delta(s))$, given in Example 3.1.
The fs-set $C_f(s)$ is an fs-semiopen set but not an fs $\mathcal{A}$-set.

Example 3.6. Example to show that an fs-semiopen set may not be fs α-open.
In the FSTS, given in Example 3.3, the fs-set $C_f(s)$ is fs-semiopen but not fs α-open.

Example 3.7. Example to show that an fs-preopen set may not be fs α-open.
Consider the fs-sets $A_f(s)$, $B_f(s)$ in a set X, defined as follows:

$$A_f(s) = \left\{\overline{\frac{1}{4}}, \overline{\frac{1}{4}}, \overline{\frac{1}{4}}, \ldots\ldots\ldots\right\}$$

$$B_f(s) = \left\{\overline{\frac{1}{5}}, \overline{\frac{1}{5}}, \overline{\frac{1}{5}}, \ldots\ldots\right\}$$

Then $\delta(s) = \{A_f(s), X_f^0(s), X_f^1(s)\}$ is a fuzzy sequential topology on X. In this FSTS, $B_f(s)$ is fs-preopen but not fs α-open.

Example 3.8. Example to show that an fs δ-set may not be fs-semiopen.
In the FSTS, given in Example 3.2, the fs-set $B_f(s)$ is an fs δ-set but is not fs-semiopen.

Example 3.9. Example to show that an fs-preopen set may not be an fs S-preopen set.
Consider the FSTS, given in Example 3.1, the fs-set $C_f(s)$ is fs-preopen but not fs S-preopen.

Example 3.10. Example to show that an fs S-preopen set may not be an fs-open set.
Consider the FSTS, given in Example 3.7, the fs-set $B_f(s)$ is fs S-preopen but not fs-open.

Definition 3.2. A mapping $g : (X, \delta(s)) \to (Y, \eta(s))$ is called
(i) fs α-continuous if $g^{-1}(B_f(s))$ is fs α-open in X, for every fs-open set $B_f(s)$ in Y.
(ii) fs lc-continuous if $g^{-1}(B_f(s))$ is locally fs-closed in X, for every fs-open set $B_f(s)$ in Y.
(iii) fs $\mathcal{A}$-continuous if $g^{-1}(B_f(s))$ is an fs $\mathcal{A}$-set in X, for every fs-open set $B_f(s)$ in Y.
(iv) fs δ-continuous if $g^{-1}(B_f(s))$ is an fs δ-set in X, for every fs-open set $B_f(s)$ in Y.
(v) fs S-precontinuous if $g^{-1}(B_f(s))$ is fs S-preopen in X, for every fs-open set $B_f(s)$ in Y.

Theorem 3.1. *An fs-set in an FSTS, is fs α-open if and only if it is fs-semiopen and fs-preopen.*

Proof. Let $A_f(s)$ be an fs α-open set, that is, $A_f(s) \leq int\ cl\ intA_f(s)$. Clearly, $A_f(s)$ is fs-semiopen. Also,

$$\begin{aligned} intA_f(s) \leq clA_f(s) &\Rightarrow cl\ intA_f(s) \leq clA_f(s) \\ &\Rightarrow int\ cl\ intA_f(s) \leq int\ clA_f(s) \\ &\Rightarrow A_f(s) \leq int\ clA_f(s) \end{aligned}$$

Thus, $A_f(s)$ is fs-preopen.

Conversely, suppose $A_f(s)$ be fs-semiopen and fs-preopen, that is, $A_f(s) \leq cl\ intA_f(s)$, $A_f(s) \leq int\ clA_f(s)$. Then,

$$\begin{aligned} & int\ clA_f(s) \leq clA_f(s) \leq cl\ intA_f(s) \\ \Rightarrow\ & A_f(s) \leq int\ cl\ intA_f(s) \end{aligned}$$

Hence, $A_f(s)$ is fs α-open. □

Corollary 3.1. *A mapping $g : (X, \delta(s)) \to (Y, \eta(s))$ is fs α-continuous if and only it is fs-semicontinuous and fs-precontinuous.*

Definition 3.3. Let $A_f(s)$ be an fs-set in an FSTS $(X, \delta(s))$. Then αfs-closure ${}_\alpha clA_f(s)$ and αfs-interior ${}_\alpha intA_f(s)$ of $A_f(s)$ are defined by

$$\begin{aligned} {}_\alpha clA_f(s) &= \wedge\{V_f(s);\ A_f(s) \leq V_f(s) \text{ and } V_f^c(s) \in \alpha(X)\} \\ {}_\alpha intA_f(s) &= \vee\{U_f(s);\ U_f(s) \leq A_f(s) \text{ and } U_f(s) \in \alpha(X)\} \end{aligned}$$

Complement of an fs α-open set is called an fs α-closed set. Hence, it is clear that ${}_\alpha cl(A_f(s))$ is the smallest fs α-closed set containing $A_f(s)$ and ${}_\alpha int(A_f(s))$ is the largest fs α-open set contained in $A_f(s)$. Further,

(i) $A_f(s) \leq {}_\alpha cl(A_f(s)) \leq \overline{A_f(s)}$ and $\overset{o}{A}_f(s) \leq {}_\alpha int(A_f(s)) \leq A_f(s)$.
(ii) $A_f(s)$ is fs α-open if and only if $A_f(s) = {}_\alpha int(A_f(s))$
(iii) $A_f(s)$ is fs α-closed if and only if $A_f(s) = {}_\alpha cl(A_f(s))$
(iv) $A_f(s) \leq B_f(s) \Rightarrow {}_\alpha int(A_f(s)) \leq {}_\alpha int(B_f(s))$ and ${}_\alpha cl(A_f(s)) \leq {}_\alpha cl(B_f(s))$.

Theorem 3.2. *Let $A_f(s)$ be an fs-set in an FSTS $(X, \delta(s))$. Then,*
(i) *${}_\alpha intA_f(s) = A_f(s) \wedge int\ cl\ intA_f(s)$.*
(ii) *if $A_f(s)$ is both fs-preopen and fs-preclosed, then $A_f(s) = int\ clA_f(s) \wedge A_f(s)$ and thus $A_f(s)$ is fs S-preopen;*
(iii) *if $A_f(s) = U_f(s) \wedge V_f(s)$, where $U_f(s)$ is fs-open and $intV_f(s)$ is fs-regular open, then ${}_\alpha intA_f(s) = intA_f(s)$;*
(iv) *if $A_f(s)$ is an fs δ-set, then ${}_\alpha intA_f(s) = {}_p intA_f(s)$.*

Proof. (i) Easy to prove.

(ii) Given $A_f(s) \leq int\ clA_f(s)$ and $cl\ intA_f(s) \leq A_f(s)$. Then,

$$A_f(s) = int\ clA_f(s) \wedge A_f(s).$$

Since $intA_f(s) = int\ cl\ intA_f(s)$, hence $A_f(s)$ is fs S-preopen.

(iii) We have $int\ cl\ intA_f(s) \leq int\ cl\ intV_f(s) = intV_f(s)$. Therefore,

$$\begin{aligned} {}_\alpha intA_f(s) &= A_f(s) \wedge int\ cl\ intA_f(s) \\ &\leq A_f(s) \wedge intV_f(s) \\ &= intA_f(s) \end{aligned}$$

Also, $intA_f(s) \leq {}_{\alpha}intA_f(s)$. Hence $intA_f(s) = {}_{\alpha}intA_f(s)$.

(*iv*) Given $int\ clA_f(s) \leq cl\ intA_f(s)$. Since ${}_{\alpha}intA_f(s)$ is an fs-preopen set contained in $A_f(s)$, we have

$$ {}_{\alpha}intA_f(s) \quad \leq \quad {}_{p}intA_f(s) $$

Now,

$$ {}_{p}intA_f(s) \leq int\ clA_f(s) \leq int\ cl\ intA_f(s) $$

Thus, ${}_{p}intA_f(s) \leq A_f(s) \wedge int\ cl\ intA_f(s) = {}_{\alpha}intA_f(s)$. Hence the result. □

Lemma 3.1. *An fs-set $A_f(s)$ is locally fs-closed if and only if $A_f(s) = U_f(s) \wedge cl(A_f(s))$, where $U_f(s)$ is an fs-open set.*

Proof. Omitted. □

Theorem 3.3. *Let $A_f(s)$ be an fs-set in an FSTS $(X, \delta(s))$. Then $A_f(s)$ is an fs $\mathcal{A}$-set if it is fs-semiopen and locally fs-closed.*

Proof. Suppose $A_f(s)$ be fs-semiopen and locally fs-closed. Then, $A_f(s) \leq cl\ intA_f(s)$ and $A_f(s) = U_f(s) \wedge clA_f(s)$, where $U_f(s)$ is fs-open. Since $clA_f(s) = cl\ intA_f(s)$ is fs-regular closed, the result follows. □

Corollary 3.2. *A mapping $g : (X, \delta(s)) \to (Y, \eta(s))$ is fs $\mathcal{A}$-continuous if it is fs-semicontinuous and fs lc-continuous.*

Remark. Unlike in a general topological space, the converse of Theorem 3.3 may not be true and it has been shown by the next Example.

Example 3.11. Let $X = \{x, y\}$. Consider the fs-sets $A_f(s)$, $B_f(s)$, $C_f(s)$, $D_f(s)$ and $E_f(s)$ in X, where

$A^1_f = \overline{0.3}$, $A^n_f(x) = 1$ and $A^n_f(y) = 0$ for all $n \neq 1$;
$B^1_f(x) = 0.4$, $B^1_f(y) = 0.7$, $B^n_f(x) = 0$ and $B^n_f(y) = 1$ for all $n \neq 1$;
$C^1_f = \overline{0.7}$ and $C^n_f = \overline{1}$ for all $n \neq 1$;
$D^1_f(x) = 0.6$, $D^1_f(y) = 0.3$, $D^n_f(x) = 1$ and $D^n_f(y) = 0$ for all $n \neq 1$;
$E^1_f(x) = 0.4$, $E^1_f(y) = 0.3$ and $E^n_f = \overline{0}$ for all $n \neq 1$.

Let $\delta(s) = \{A_f(s), B_f(s), C_f(s), A_f(s) \wedge B_f(s), A_f(s) \vee B_f(s), X^0_f(s), X^1_f(s)\}$. In the FSTS $(X, \delta(s))$, $D_f(s)$ being an fs-regular closed set, the fs-set $E_f(s) = B_f(s) \wedge D_f(s)$ is an fs $\mathcal{A}$-set but not fs-semiopen.

Theorem 3.4. *Let $(X, \delta(s))$ be an FSTS and $A_f(s)$ be an fs-set in X. Then the following statements are equivalent:*
(i) $A_f(s)$ is an fs-open set.
(ii) $A_f(s)$ is fs α-open and locally fs-closed.
(iii) $A_f(s)$ is fs-preopen and locally fs-closed.
(iv) $A_f(s)$ is fs-preopen and an fs $\mathcal{A}$-set.
(v) $A_f(s)$ is fs S-preopen and an fs δ-set.

Proof. $(i) \Rightarrow (ii)$ and $(ii) \Rightarrow (iii)$ are obvious.

$(iii) \Rightarrow (iv)$ Let $A_f(s)$ be fs-preopen and locally fs-closed. Then,

$$A_f(s) \leq int\ clA_f(s) \quad \text{and} \quad A_f(s) = U_f(s) \wedge clA_f(s),$$

where $U_f(s)$ is fs-open. $clA_f(s)$ being fs-regular closed, the result follows.

$(iv) \Rightarrow (i)$ Let $A_f(s)$ be an fs-preopen and an fs $\mathcal{A}$-set. Then,

$$A_f(s) \leq int\ clA_f(s) \quad \text{and} \quad A_f(s) = U_f(s) \wedge clA_f(s),$$

where $U_f(s)$ is fs-open. Since $intA_f(s) = U_f(s) \wedge int\ clA_f(s)$, $A_f(s)$ is fs-open.

$(i) \Rightarrow (v)$ Obvious.

$(v) \Rightarrow (i)$ Let $A_f(s)$ be an fs S-preopen and an fs δ-set. Using Theorem 3.2, (iii) and (iv),

$$intA_f(s) = {}_{\alpha}intA_f(s) = {}_{p}intA_f(s) = A_f(s).$$

Hence, $A_f(s)$ is fs-open. □

By Theorems 3.1, 3.3 and 3.4, we have the following relationships among the different classes of fs-sets of an FSTS $(X, \delta(s))$:
(i) $\alpha(X) = FSPO(X) \cap FSSO(X)$.
(ii) $\mathcal{A}(X) \supseteq FSSO(X) \cap FSLC(X)$.
(iii) $\delta(s) = \alpha(X) \cap FSLC(X)$.
(iv) $\delta(s) = FSPO(X) \cap FSLC(X)$.
(v) $\delta(s) = FSPO(X) \cap \mathcal{A}(X)$.
(vi) $\delta(s) = FSSPO(X) \cap \delta(X)$.

Theorem 3.5. *In an FSTS $(X, \delta(s))$, the following are equivalent:*
(i) $clA_f(s) \in \delta(s)$ for every $A_f(s) \in \delta(s)$.
(v) $\mathcal{A}(X) = \delta(s)$.

Proof. $(i) \Rightarrow (ii)$ It is obvious that $\delta(s) \subseteq \mathcal{A}(X)$. For the reverse inclusion, let $A_f(s) \in \mathcal{A}(X)$, then

$$A_f(s) = U_f(s) \wedge V_f(s),$$

where $U_f(s)$ is fs-open and $V_f(s)$ is fs-regular closed. By (i), $V_f(s) \in \delta(s)$ and hence $A_f(s) \in \delta(s)$.

$(ii) \Rightarrow (i)$ Suppose $\mathcal{A}(X) = \delta(s)$. Let $A_f(s) \in \delta(s)$, then $clA_f(s)$ is fs-regular closed and hence belongs to $\mathcal{A}(X) = \delta(s)$. □

We conclude the chapter by stating the following decompositions of fs-continuity:

Theorem 3.6. *Let $g : (X, \delta(s)) \to (Y, \eta(s))$ be a map. Then g is fs-continuous if and only if*
(i) *g is fs α-continuous and fs lc-continuous.*
(ii) *g is fs-precontinuous and fs lc-continuous.*
(iii) *g is fs-precontinuous and fs $\mathcal{A}$-continuous.*
(iv) *g is fs S-continuous and fs δ-continuous.*

References

Azad, K. K. (1981). On fuzzy semicontinuity, fuzzy amost continuity and fuzzy weakly continuity. *J. Math. Anal. Appl.* **82**, 14–32.

Mashhour, A. S., M. E. Abd El-Monsef and S. N. EI-Deeb (1982). On precontinuous and weak precontinuous mappings. *Proc. Math. Phys. Soc. Egypt* **53**, 47–53.

Singha, M., N. Tamang and S. De Sarkar (2014). Fuzzy sequential topological spaces. *International Journal of Computer and Mathematical Sciences* **3**(4), 2347–8527.

Tamang, N. and S. De Sarkar (2016). Some nearly open sets in a fuzzy sequential topological space. *International Journal of Mathematical Archive* **7**(2), 120–128.

Tamang, N., M. Singha and S. De Sarkar (2016). On the notions of continuity and compactness in fuzzy sequential topological spaces. *International Journal of Engineering and Advanced Research Technology* **2**(10), 9–15.