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An Application of Fuzzy Sets to Veterinary Medicine

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Abstract

In this paper, firstly, the waves P and T in ECG of kittens and adult cats were converted to fuzzy sets. After, using to entropy definition for fuzzy sets, we have assigned an entropy to waves P and T for kittens and adult cats. Also, using to some new formulates, the graphical representation of waves P and T for normal or diseased heart of cats were given.

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1. Introduction

The theoretical and practical applications of fuzzy sets have increased considerably since Zadeh's paper, (see (Abdollahian *et al.*, 2010) ; (Bilgin, 2003); (Dhar, 2013); (Diamond & Kloeden, 1994); (Goetschel & Voxman, 1986); (Li *et al.*, 1995); (Iwamoto & et al, 2007); (Kosko, 1986); (Matloka, 1986); Tong *et al.* (2007); (Zadeh, 1965) and (Zararsız & Şengönül, 2013)). In medicine, cardiologists are try to predetermine some heart diseases from electrocardiographs and this processes is also valid for veterinary medicine. Some fine details may not be seen in graphical representation of the waves electrocardiographs of human or animals. It is a fact that, long time can be spent for interpreting electrocardiographs (shortly; ECG) and sometimes small but important details can be unnoticed or ECG's can be misleading for junior vet or cardiologists. In this paper, by using entropy concept, we have obtained numerical values for ECGs of kittens and adult cats. These numerical values are the best way to observe fine details in the waves such as P, PQR complex and T. The numerical values are also very clear and can be easily interpreted for any person according to graphical representation of ECG's. It will be seen that these computations are

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completely different than computations of (Czogala & Leski, 2000). Let us give some background information on fuzzy sets and entropy of the fuzzy sets.

Let $\mathscr{X}$ be nonempty set. According to Zadeh, a fuzzy subset of $\mathscr{X}$ is a nonempty subset $\{(x, u(x)) : x \in \mathscr{X}\}$ of $\mathscr{X} \times [0, 1]$ for some function $u : \mathscr{X} \to [0, 1]$, (Diamond & Kloeden, 1994). Consider a function $u : \mathbb{R} \to [0, 1]$ as a subset of a nonempty base space $\mathbb{R}$. The function u is called membership function of the fuzzy set u.

Furthermore, we know that shape similarity of the membership functions does not reflect the conception of itself, but it will be used for examining the context of the membership functions. Whether a particular shape is suitable or not can be determined only in the context of a particular application. However, that many applications are not overly sensitive to variations in the shape. In such cases, it is convenient to use a simple shape, such as the triangular shape of membership function. Let us define fuzzy set u on the set $\mathbb{R}$ with membership function as follows:

$$u(x) = \begin{cases} \frac{h_u}{u_1 - u_0}(x - u_0), & x \in [u_0, u_1) \\ \frac{-h_u}{u_2 - u_1}(x - u_1) + h_u, & x \in [u_1, u_2] \\ 0, & others \end{cases}, \tag{1.1}$$

where the notations h_u denotes height of the fuzzy sets u. For brief, we write triple $(u_0, u_1 : h_u, u_2)$ for fuzzy set u. Notation $\mathscr{F}$ be the set of the all fuzzy sets in the form $u = (u_0, u_1 : h_u, u_2)$ on the $\mathbb{R}$.

Define the function S as follows:

$$S : \mathscr{F} \times \mathscr{F} \to \mathbb{R}, \quad S(u, v) = \frac{\min\{h_u, h_v\}}{\max\{h_u, h_v\}}[1 - \frac{1}{3}\sum_{k=0}^{2} |u_k - v_k|]. \tag{1.2}$$

The function S is called similarity degree between the fuzzy sets u and v. If $S(u, v) = 1$ then we say that u is full similar to v or vice versa, we say that v is completely similar to u. If $0 < S(u, v) < 1$ then we say that the fuzzy set u is S- similar to the fuzzy set v (or the fuzzy set v is S- similar to the fuzzy set u), if $S(u, v) \leq 0$ we say that, u is not similar to v. Similar definitions can be found in (Sridevi & Nadarajan, 2009) and (Yıldız & Şengönül, 2014).

If we capture numerous ECG for any human or animal, it can be considered as a finite sequence of ECG's. Therefore we will give some definitions and properties about sequences of fuzzy sets.

The set

$$w(\mathscr{F}) = \{(u_k) \mid u : \mathbb{N} \to \mathscr{F}, u(k) = (u^k) = ((u_0^k, u_1^k : h_{u^k}, u_2^k))\} \tag{1.3}$$

is called sequence of fuzzy sets. Any element of the set $w(\mathscr{F})$ is called sequences of fuzzy sets, where $u_0^k, u_1^k, u_2^k \in \mathbb{R}$, $u_0^k \leq u_1^k \leq u_2^k$ and the mean of notation $u_1^k : h_{u^k}$ is the k^{th} therm of the sequence (u^k) takes highest membership degree at u_1^k and this membership degree is equal to h_{u^k} . If for all $k \in \mathbb{N}$, $h_{u^k} = 1$ then the set $w(\mathscr{F})$ turns into sequence set of fuzzy numbers and if $u_0^k = u_1^k = u_2^k$ and $h_{u_1^k} = 1$ the set $w(\mathscr{F})$ turns in to ordinary sequence space of the real numbers, respectively.

An another important class of the sequence set of the fuzzy sets is defined by

$$\varphi(\mathscr{F}) = \{(u_k) \in w(\mathscr{F}) \mid \exists k_0 \in \mathbb{N}, \forall k \geq k_0 : u^k = 0\}. \tag{1.4}$$

Clearly, the sequences of fuzzy sets can obtain by fuzzification of the term by term of sequence of real numbers with a suitable method.

Definition 1.1. Let us define the function $\mathscr{S}$ as follows:

$$\mathscr{S}: w(\mathscr{F}) \times w(\mathscr{F}) \to \mathbb{R}, \quad \mathscr{S}(u_n, v_n) = \frac{\inf\{h_{u_n}, h_{v_n}\}}{\sup\{h_{u_n}, h_{v_n}\}}[1 - \frac{1}{3} \lim_n \sum_{k=0}^{2} |u_k^n - v_k^n|] = \lambda. \tag{1.5}$$

The function $\mathscr{S}$ is called similarity degree between sequences of fuzzy sets (u_n) and (v_n). If $\mathscr{S}(u_n, v_n) = 1$ then we say that (u_n) is completely similar to the sequence (v_n), if $0 < \mathscr{S}(u_n, v_n) = \lambda < 1$ then we say that the sequence (u_n) is λ- similar to the sequence (v_n), if $\lambda \leq 0$ we say that, (u_n) is not similar to (v_n).

In the fuzzy set theory, the fuzziness of a fuzzy set is a important matter and there are many method to measure the fuzziness of a fuzzy set. At first, the fuzziness was thought to be the distance between fuzzy set and its nearest nonfuzzy set. Later, the entropy was used instead of of fuzziness (de Luca & Termini, 1972) and has received attention, recently (Wang & Chui, 2000). Well, then what is the entropy?

Definition 1.2. (Zimmermann, 1991) Let $u \in \mathscr{F}$ and $u(x)$ be the membership function of the fuzzy set u and consider the function $H : \mathscr{F} \to \mathbb{R}^+$. If the function H satisfies conditions below,

1. $H(u) = 0$ iff u is crisp set,
2. $H(u)$ has a unique maximum, if $u(x) = \frac{1}{2}$, for all $x \in \mathbb{R}$
3. For $u, v \in \mathscr{F}$, if $v(x) \leq u(x)$ for $u(x) \leq \frac{1}{2}$ and $u(x) \leq v(x)$ for $u(x) \geq \frac{1}{2}$ then $H(u) \geq H(v)$,
4. $H(u^c) = H(u)$, where u^c is the complement of the fuzzy set u

then the $H(u)$ is called entropy of the fuzzy set u.

Let suppose that $u = u(x)$ be membership function of the fuzzy set u and the function $h : [0, 1] \to [0, 1]$ satisfies the following properties:

1. Monotonically increasing at $[0, \frac{1}{2}]$ and decreasing $[\frac{1}{2}, 1]$,
2. $h(x) = 0$ if $x = 0$ and $h(x) = 1$ if $x = \frac{1}{2}$.

The function h is called entropy function and the equality $H(u(x)) = h(u(x))$ holds for $x \in \mathbb{R}$. Some well known entropy functions are given as follows:

$h_1(x) = 4x(1 - x)$, $h_2(x) = -x \ln x - (1 - x) \ln(1 - x)$, $h_3(x) = \min\{2x, 2 - 2x\}$ and

$$h_4(x) = \begin{cases} 2x, & x \in [0, \frac{1}{2}] \\ 2(1 - x), & x \in [\frac{1}{2}, 1] \end{cases}.$$

Note that the function h_1 is the logistic function, h_2 is called Shannon function and h_3 is the tent function.

Let $\mathscr{X}$ be a continuous universal set. The total entropy of the fuzzy set u on the $\mathscr{X}$ is defined

$$e(u) = \int_{x \in \mathscr{X}} h(u(x)) p(x) dx \tag{1.6}$$

where $p(x)$ is the probability density function of the available data in $\mathscr{X}$ (Pedrycz, 1994), (Pedrycz & Gomide, 2007). If we take $p(x) = 1$ in the (1.6) then the $e(u)$ is called entropy of the fuzzy set

u. It is known that the value of $e(u)$ is depend on support of the fuzzy set u. Let u be fuzzy set on the set $\mathbb{R}$ with membership function (1.1), then we see that the total entropy of fuzzy set u is equal to

$$e(u) = c(2h_u - \frac{4}{3}h_u^2)\ell(u) \tag{1.7}$$

for $p(x) = c$ and $h = h_1$, where $\ell(u) = \max\{x - y : x, y \in \overline{\{x \in \mathbb{R} : u(x) > 0\}}\}$. We know that each fuzzy set or a fuzzy number correspond to the fuzzy thoughts in the idea of user. So, any sequence of the fuzzy sets can be seen as sequence of thoughts or sequence fuzzy information. This sequence of fuzzy information may contain an useful information or not contain an useful information. But we can use terms of this sequence to obtain meaningful information from this sequence.

Definition 1.3. Let h be an entropy function, (u^k) be a sequence of fuzzy sets (or fuzzy thought) and $p_k(x)$ be probability density function of the available data in $\mathbb{R}$ for every $k \in \mathbb{N}$. Then sequence

$$e(u^k) = \int_{x\in\mathbb{R}} h(u^k(x))p_k(x)dx \tag{1.8}$$

is called total entropy sequence of the fuzzy sets (u^k). If the probability density function $p_k(x) = 1$ is fix, for all $k \in \mathbb{N}$, then the (1.8) is called entropy sequence of the fuzzy sets $u = (u^k)$.

If we take $u = (u^k) \in w(\mathscr{F})$, $p_k(x) = c_k \in (0, 1]$ and $h(u) = h_1(u)$ then from (1.8) we have

$$e(u^k) = (c_k(2h_{u^k} - \frac{4}{3}h_{u^k}^2)\ell(u^k)), \tag{1.9}$$

here and other places in the text, the notation $2h_{u^k}^2$ denotes second power of the h_{u^k}. If we choose the probability density functions $p_k(x) = c \in (0, 1]$ for all $k \in \mathbb{N}$ and $h_{u^k} = 1$ for all $k \in \mathbb{N}$ in the (1.9) then we see that $e(u^k) = \frac{2}{3}c\ell(u^k)$.

Let us suppose that $u = (u^k)$ be sequences of the fuzzy numbers (that is $h_{u^k} = 1$), $h(u) = h_1(u)$ and $p_k(x) = c_k = 1 \in (0, 1]$ for all $k \in \mathbb{N}$. Then the entropy $e(u^k)$ of the sequence of fuzzy numbers (u^k) is equal to

$$e(u^k) = \frac{2}{3}\ell(u^k). \tag{1.10}$$

Clearly, if $\ell(u^k) = 0$ for every $k \in \mathbb{N}$ then the sequence (u^k) returns to sequence of real numbers. In this case the entropy of the total entropy sequence is zero for sequences of real numbers. For example, let $u = (u^k)$ be $((1, 1 : 1, 1))$, then from (1.10) we obtain zeros sequence. Furthermore, the entropy sequence (e_k) can not be convergent but be bounded.

Definition 1.4. Let $\mathscr{A} = (a_{nk})$ be a lower triangular infinite matrix of real or complex numbers and

$$\sum_k a_{nk} \int_{x\in\mathbb{R}} h(u^k(x))p_k(x)dx \to E, \quad n \to \infty. \tag{1.11}$$

The real number E is called total $\mathscr{A}$-entropy of the sequence (u^k) of fuzzy sets, if it exists.

Definition 1.5. Let suppose that the $u = (u^k)$ be a sequence of fuzzy sets, $p_k(x) = c_k$, $(c_k \in (0, 1])$ for all $k \in \mathbb{N}$ and

$$\lim_n \sum_k a_{nk} \int_{x\in\mathbb{R}} h(u^k(x))p_k(x)dx = \lim_n \sum_k a_{nk}c_k(2h_{u^k} - \frac{4}{3}h^2_{u^k})\ell(u^k) = E_1. \tag{1.12}$$

The real number E_1 is called total $\mathscr{A}$- entropy according to entropy function h and $p_k(x) = c_k$ is probability density functions of the sequence $u = (u^k)$ of fuzzy sets, and it is shown by $T_e^{\mathscr{A}}(u^k)$.

Let $n, k \in \mathbb{N}$, $\alpha > -1$, $p_k(x) = c_k$ and $\binom{n-k+\alpha-1}{n-k}$, $\binom{n+\alpha}{n}$ are binomial confidence. Let us define infinite matrices $A = (a_{nk})$ and $C^\alpha = (c^\alpha_{nk})$ as follows:

$$a_{nk} = \begin{cases} 1, & 0 \le k \le n \\ 0, & otherwise \end{cases} \quad \text{and} \quad c^\alpha_{nk} = \begin{cases} \frac{\binom{n-k+\alpha-1}{n-k}}{\binom{n+\alpha}{n}}, & 0 \le k \le n \\ 0, & otherwise \end{cases}.$$

If we write the matrices A and C^α instead of $\mathscr{A}$ in the expression (1.12) then we have

$$\lim_n \sum_{k=0}^{n} \int_{x\in\mathbb{R}} h(u^k(x))p_k(x)dx = T_e^A(u^k) \tag{1.13}$$

and

$$\lim_n \frac{1}{\binom{n+\alpha}{n}} \sum_{k=0}^{n} \binom{n-k+\alpha-1}{n-k} \int_{x\in\mathbb{R}} h(u^i(x))p_k(x)dx = T_e^{C^\alpha}(u^k), \tag{1.14}$$

respectively.

The expressions (1.13) and (1.14) are called A- total entropy and total Cesàro entropy of order α of the sequence $u = (u^k)$ of fuzzy sets, according to probability density functions $p_k(x)$, respectively. If we take $\alpha = 1$ and $p_k(x) = c_k$ from (1.14) we see that

$$T_e^{C^1}(u^k) = \lim_n \frac{1}{n+1} \sum_{k=0}^{n} c_k(2h_{u_1^k} - \frac{4}{3}h^2_{u_1^k})\ell(u^k) \tag{1.15}$$

which is called Cesàro normalized entropy of order 1 (shortly, Cesàro entropy) of the sequence $u = (u^k)$ of fuzzy sets.

It is easily prove that, if

$$T_e^{C^1}(u^k) = \lim_n \frac{1}{n+1} \sum_{k=0}^{n} c_k(2h_{u_1^k} - \frac{4}{3}h^2_{u_1^k})\ell(u^k) = a$$

then

$$T_e^{C^1}(u^k) = \lim_n \frac{s}{n+r} \sum_{k=0}^{n} c_k(2h_{u_1^k} - \frac{4}{3}h^2_{u_1^k})\ell(u^k) = a$$

where $r, s \in \mathbb{R}$. For example, the Cesàro entropy of sequence $(u^k) = ((\frac{k}{k+1} - t_1, \frac{k}{k+1} : 1, \frac{k}{k+1} + t_2))$ is

$$T_e^C(u^k) = \lim_n \frac{2(t_2 + t_1)}{3(n+1)} \sum_{k=0}^{n} c_k, \tag{1.16}$$

where we assume that $t_1 < t_2$ and $t_1, t_2 \in \mathbb{R}$ and $h_{u^k} = 1$ for all $k \in \mathbb{N}$. If the series $\sum_k c_k$ is convergent then the value $T_{e^{C^1}}(u^k)$ exists every time. As a comment of the (1.13) and (1.16), we point out that we can obtain an useful information from infinite fuzzy information by a suitable method. But, the total entropy and Cesàro entropy of the sequence v defined by $v = ((v_0^k, v_1^k, v_2^k)) = ((-k, 1 : 1, k + 2))$ is infinite. This means that, the sequence v does not contain any useful information for us.

Since, every real number is also a fuzzy number then we can give following corollary:

Corollary 1.1. *Let the sequence $r = (r^k)$ be a convergent or divergent sequence of real numbers. Then the all entropies of the $r = (r^k)$ are zero.*

Corollary 1.1 can be interpreted as, in the any information sequence, if the elements of information sequence are crisp information then we obtain a crisp information from this sequence.

Proposition 1.1. If the fuzziness of the any sequence of fuzzy set is constantly increasing then the entropy is constantly grow and maybe is infinite. On the contrary if the fuzzyness of the any sequence of fuzzy set is constantly decreasing then the entropy is decreases and becomes 0.

It is calculated in (Chin, 2006) that the entropy of any fuzzy number is $\frac{2c(u_2-u_0)}{3}$. Therefore, in generally, if we take $h = h_1$ and $p_i(x) = c$, for every $i \in \mathbb{N}$, then entropy of the sequence of fuzzy numbers is given with (1.10).

In next section, we will investigate entropy of the electrocardiogram for cats and give some comments. We know that, an electrocardiogram is an important test for any relevant heart diseases of human or animals, the shortest way of identifying heart problems and you can detects cardiac (heart) abnormalities, as an example heart attacks, an enlarged heard or abnormal heart rhythms may cause heart failure, abnormal position of heart can be given, by measuring the electrical activity generated by the heart as it contacts, (for more, see (de Luna, 1987)).

2. The Applications to ECG's of the Idea Entropy and Some Comments

It is a fact that, the long time can be spent for interpreting electrocardiographs results by cardiologists or vet and sometimes small but important details can be unnoticed because of complexity of the *ECG*. The same situation is also valid for computerized electrocardiography. According to us, numerical values for *ECG* outputs can be more reliable for cardiologists and vet for interpreting *ECG* results. Furthermore, if the outputs are numerical then the consultation may be easy than consultation of the *ECG* papers. In this section we have proposed a new consultation method for cardiac problems of cats which will be based upon numerical value of *ECG*s, (see (Brady & Rosen, 2005); (Khan, 2003) for ECG).

Quite simply every heart beats can be considered as term of a sequence. Using to the waves P, QRS complex and T, we can construct the waves sequence $((P_k, (QRS)_k), T_k))$, where k is beat number or number of measurements and is finite. The graphical shapes of the waves P, QRS complex and T can imagine a membership functions a fuzzy set. With this idea, we can appoint an entropy value using to these membership functions which will be described below.

The entropy of the sequence $((P_k, (QRS)_k), T_k))$ can compute for finite or infinite many k and this computation gives to us a numerical value, not graphical. From numerical value, we can determine some cardiac problems. Namely, the sequence $((P_k, (QRS)_k), T_k))$ can divide three part for calculate entropy as follows:

1. The entropy of the sequence (P_k) waves,
2. The entropy of the sequence $((QRS)_k))$ complexes,
3. The entropy of the sequence (T_k) waves.

In this case, we can assume that the total entropy of the heart is equal to

$$\mathscr{E} = e(P_k) + e((QRS)_k) + e(T_k). \tag{2.1}$$

Now we will summarize some information about electrocardiographs without deepening the subject.

The electrocardiograph records the electrical activity of the heart muscle and displays this data as a trace on a screen or on paper and, later, this data is interpreted by a medical practitioner. ECG's from healthy hearts have a characteristic shape. Any irregularity in the heart rhythm or damage to the heart muscle can change the electrical activity of heart which leads to change in the shape of ECG's according to patients. Using this changes, we can investigate entropy of the heart rhythm or damage entropy of the heart muscle. It is known that, the QRS complex reflect the rapid depolarization of the right and left ventricles. The ventricles have a large muscle mass compared to the atria so the QRS complex usually has a much larger amplitude than the P- wave.

Furthermore, the heart movements are kept in check by various charges and pulses that change slightly on exertion, blood chemistry and strain. According to us, residence of skin and conductivity of blood are important for ECG, too. The conductivity and residence of the skin are vary according to some minerals in the blood plasma such as calcium, chloride, potassium or glucose concentration in a diabetic patients blood. So we have to consider the conductivity of blood in the calculations of transmitting electric current and therefore in the entropy calculations for a heart. For blood conductivity properties, you can read to (Hirsch & et al, 1950).

2.1. *The Entropy of The Waves Sequence (P_k) and Some Comments*

Primary wave of a heart in ECG, is called P wave and shortly denoted with P, have an entropy value and it can be compute as follows:

$$e(P) = \int_{x\in\mathbb{R}} h_1(P(x))r(x)dx, \tag{2.2}$$

where the function $P(x)$ is membership function of the fuzzy $\mathscr{P}$ set that we will correspond to wave P and the function $r(x)$ is conductivity function (generally the function r is fix) of the body .

Experimental measurements showed that to us for kittens, the wave P has maximal height about $0.12mV$, duration is shorter than 0.3 seconds but these values for adult cats are $0.2mV$ second and 0.04 (Lourenço & Ferreira, 2003).

Using the maximal height and duration of wave P as 0.12 second and 0.3 mV, respectively, the membership function $P_1(x)$ of the fuzzy $\mathscr{P}_1$ set which is correspond to wave P for kittens can write as follows:

$$P_1(x) = \begin{cases} 0.8x, & x \in [0, 0.15] \\ 0.24 - 0.8x, & x \in (0.15, 0.30] \\ 0, & \text{otherwise} \end{cases}. \tag{2.3}$$

Furthermore, the membership function $P_2(x)$ of the fuzzy $\mathscr{P}_2$ set which is correspond to wave P for adult cats is

$$P_2(x) = \begin{cases} 10x, & x \in [0, 0.02] \\ 0.4 - 10x, & x \in (0.02, 0.04] \\ 0, & \text{otherwise} \end{cases}. \tag{2.4}$$

It is clear that the support of the fuzzy set $\mathscr{P}_1$ is duration of the wave P and height is maximum height of wave P.

Let us take $supp\mathscr{P}_1 \approx]0, 0.30[$, $supp\mathscr{P}_2 \approx]0, 0.04[$ and closure of the $supp\mathscr{P}_1$ and $supp\mathscr{P}_2$ be $\overline{supp\mathscr{P}_1} = [0, 0.30]$ and $\overline{supp\mathscr{P}_2} = [0, 0.04]$ where the notations $supp\mathscr{P}_1$ and $supp\mathscr{P}_2$ denotes support of the $\mathscr{P}_1$ and $\mathscr{P}_2$.

In this case, we see that $h_1(P_1(x)) = \begin{cases} 3.2x - 2.56x^2, & x \in [0, 0.15] \\ 0.7296 - 1.664x - 2.56x^2, & x \in (0.15, 0.30] \\ 0, & \text{otherwise} \end{cases}$. Similarly to $h_1(P_1(x))$, we have $h_1(P_2(x)) = \begin{cases} 40x - 400x^2, & x \in [0, 0.02] \\ 0.96 - 8x - 400x^2, & x \in (0.02, 0.04] \\ 0, & \text{otherwise} \end{cases}$.

Let us denote P_1 and P_2 of wave P for kittens and adult cats, respectively. If we choose $r(x) = c$ in (2.2) then we see that the the entropy of wave P_1 is equal to

$$e(P_1) = 662.4 \times 10^{-4} c \tag{2.5}$$

for normal wave P for kittens. The P_2 wave entropy for adult cats is

$$e(P_2) = 138.667 \times 10^{-4} c. \tag{2.6}$$

If we compare (2.5) and (2.6) then we see that the P wave entropies of the kittens and adult cats are different.

Definition 2.1. The total Cesàro entropy of the sequence (P_k) is

$$T_e^{C^1}(P_k) = \frac{1}{k+1} \sum_{i=0}^{k} c_i a_2^i (2h_{a_1^i} - \frac{4}{3} h_{a_1^i}^2) S(P_k, P), \tag{2.7}$$

where c_i is resistance of the dry skin in the ith sample, k is number of sample of P wave and $S(P_k, P)$ is similarity degree between of the waves P_k and P.

Table 1. Non-clinical P waves data for adult cats

Gender: Male	Age:xx	Weight:xx	Height:xx							
Days	1	2	3	4	5	6	7	8	9	10
$m(h_{a_k^1})$	0.2	0.2	0.19	0.21	0.23	0.23	0.19	0.2	0.18	0.15
$m(a_2^k)$	0.04	0.04	0.03	0.05	0.05	0.05	0.045	0.044	0.043	0.043
$e(P_k)$	0,0138672	0,0138672	0,009956361	0,018060735	0,019474215	0,019474215	0,017526794	0,014602663	0,013622864	0,011610323
$S(P_k, P)$	1	1	0,94525	0,947619048	0,865217391	0,865217391	0,867391304	0,9481	0,89865	0,748875

Let the resistance of the dry skin be fix that is if c_i equal to c at the each every i. place then the (2.7) is turn to

$$T_e^{C^1}(P_k) = \frac{c}{k+1}\sum_{i=0}^{k} a_2^i(2h_{a_1^i} - \frac{4}{3}h_{a_1^i}^2)S(P_k, P). \tag{2.8}$$

Example 2.1. Let us suppose, the wave P values as height and width as given in Table 1 for any adult cat for 10 measurements with fix conductivity of blood and residence of the skin. Note that these data are not clinical measures. In this mean, the sequence (P_k) is in the set $\varphi(\mathscr{F})$. The notations $m(h_{a_k^1})$ and $m(a_2^k)$ in Table 1 denotes measured height and durations of the wave P in day. Then from (2.8), we see that the Cesàro total entropy of the wave P of adult cats according to Table 1 is

$$T_e^{C^1}(P_k) = 137.94345 \times 10^{-4} c \tag{2.9}$$

for 10 beats. If we compare (2.5) and (2.9), the P wave properties of the adult cat heart which given above example is very low than normal value. Using to (1.7), we can give a graphic for 10 sample of wave P which given in the Table 1 (see, Figure 2) .

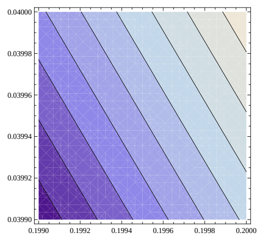

Figure 1
Graphical representation of $e(P_k)$ of the normal P wave for adult cats.

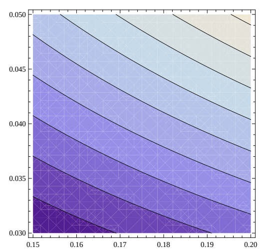

Figure 2
Graphical representation of $e(P_k)$ for Table 1 values for adult cats.

The Figure 1 is entropy graphic for the normal wave P of adult cats. If we compare the Figures 1 and 2 then we see that the height and duration of the P wave when changed with any effect, the all entropy zones are curl to upward at adult cats as in humans. It can be consider that the magnitude of the curl is P wave degenerations.

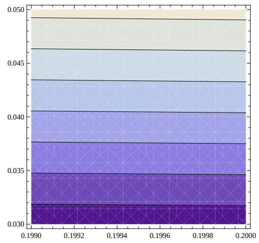

Figure 3
The values $h_{a_k^2}$ nearly fix but values a_2^k variable for adult cats.

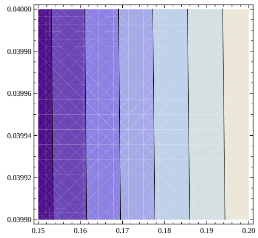

Figure 4
The values a_2^k nearly fix but the values $h_{a_k^2}$ variable for adult cats.

If the $h_{a_k^1}$ is fix but the value a_2^k be variable and conversely the $h_{a_k^1}$ is variable but the value a_2^k be fix then graphical representation of the entropy zones are shown as in Figure 3 and Figure 4, respectively.

As similar to (2.7), the A- entropy of the sequence wave P is

$$T_e^A(P_k) = 1379.43446 \times 10^{-4}c \tag{2.10}$$

from (1.13). But normal A-entropy value for 10 beats of adult cats should be $6624 \times 10^{-4}c$ and the P wave value in (2.10) very low than $6624 \times 10^{-4}c$. where c is resistance of the dry skin in the i^{th} time.

Comment 1.

We know that the value of the $S(P_k, P)$ must be $0 \leq S(P_k, P) \leq 1$ for every $k \in \mathbb{N}$. After a certain place, if P_k waves is not exists, or the similarity values $S(P_k, P)$ nearly to the zero then the entropy of atrial depolarization of the heart, the $T_e^A(P_k)$ is near to zero. In this case we can say that this is a risk (for example, it can indicate hyperkalemia or hypokalemia or right atrial enlargement for this heart in the future as in human.

Comment 2.

Respectively, if the values $e(P_1)$ and $e(P_2)$ less than $662.4\times10^{-4}c$ and $138.667\times10^{-4}c$ for kitten and adult cats then, we can say that, there is a risk (for example, it can indicate hyperkalemia or hypokalemia or right atrial enlargement as in human for this heart in the future.

3. Comparison with the ECG

1. Long time can be spent for interpreting electrocardiographs results by cardiologists or vets and sometimes small but important details can be unnoticed because of the complexity of ECG.
2. Numerical values are more reliable than graphical representations.
3. If the outputs are numerical then the consultation may be easy than consultation of the ECG papers.

4. Weakness of This Model

The weakness of this model is that the data may be incomplete and not accurate enough because of the system that we use when we collect the data. Kittens adaptation to ECG machines is an important factor in the measurement phase since heart rates can change under stress and different circumstances. The numerical values may not reflect the reality if the information is not in the near proximity of real world assessment, shortly wrong inputs can produces misleading results.

5. Conclusions and Suggestions

The conclusions can be summarized as follows:

1. The entropy of the wave P for normal heart of the kitten should be $1379.43446 \times 10^{-4}c$ and should be $6624 \times 10^{-4}c$ for adult cats.
2. The graphical representation of the normal wave P of kittens should similar to Figure 1.
3. If the duration is fix but height is being altered by any reason then lines in graphical representation of the wave P becomes steeper.
4. The lines in the graphical representation of the wave T should be almost parallel to horizontal axis.

As a suggestion, clearly, one can define entropy value and graphical representations of QRS complex and wave T to similar entropy value wave P. So any numerical value can obtain for (2.1). If entropy value of the QRS complex and wave P are calculate then we can give a numerical entropy value for (2.1).

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