Theory and Applications of
Mathematics & Computer Science

(ISSN 2067-2764, EISSN 2247-6202)
http://www.uav.ro/applications/se/journal/index.php/tamcs

Theory and Applications of Mathematics & Computer Science 5 (1) (2015) 20–28

Logical Tree of Mathematical Modeling

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Abstract

During setting up a mathematical model, it can be very important and difficult task to choose input parameters that should be known for solution of this problem. A similar problem might come up when someone wants to carry out an engineering calculation task. A very essential aim technical education is developing of good logical engineering thinking. One main part of this thinking is to determine the potential sets of required input parameters of an engineering calculation. This paper proposes a logical tree based method to determine the required parameters of a mathematical model. The method gives a lively description about needed data base, and computational sequence for us to get to determine the set of required output parameter. The shown method is named **LogTreeMM** - **Log**ical **Tree** of **M**athematical **M**odeling.

Keywords: mathematical modeling, logical tree, engineering thinking, STEM education.
2010 MSC: 93A30, 00A71, 97D30.

1. Introduction

On the one hand, during engineering work, dozens and dozens of times we should determine any parameters of a technical system or process. To carry out the task mentioned above we have to know some input parameters of the investigated system or process. We can meet similar task during setting up a mathematical model for system or process simulation. The correct identification of optimal set of required input parameters is an important and hard engineering task.

On the other hand, during technical education it is a very difficult and essential task to develop good logical engineering thinking of students or pupils. One main part of this thinking is to determine the optimal set of required input parameters of the calculation task mentioned above.

The main aim of this paper is to show a logical tree method to determine required parameters of a mathematical model or an engineering calculation. This method gives a lively description about needed data base, and computational sequence for us to get to determine the required output

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parameter. As the flow chart emphasizes the main steps of a calculation task, the proposed logical tree demonstrates the interdependencies and interrelations of variables. To choose a set of required parameters, firstly we should have an applicable equation to calculate the output parameter of the system that is a dependent variable of its model. Knowing this adaptable equation we can face the next question: How can we determine the independent variable(s) of the foregoing equation? And we should ask it repeatedly. ... It is possible that we can furnish two or more answers to one of these questions. In this case we get different required model parameter(s).

It is easily statable that we use logical inferences during determination of needed parameters to set up and to apply a mathematical model. In the one hand we use AND logical operations if all parameters should be known. On the other hand we use OR logical operations if we know two or more equations to calculate a parameter.

These logical connections are used in Fault Tree Analysis to determine causes of a system failure. The handbooks of NASA (Stamatelatos & Caraballo, 2002) and U.S. Nuclear Regulatory Commission (Vesely *et al.*, 1987) show theoretical background and practical questions of FTA. There are several publications that propose reliability or safety methods based on FTA. For example, Tchorzewska-Cieslak and Boryczko presented the methodology of the FTA and an example of its application in order to analyze different failure scenarios in water distribution subsystem (Tchorzewska-Cieslak & Boryczko, 2010). They concluded that the FTA is particularly useful for the analysis of complex technical systems in which analysis of failure scenarios is a difficult process because it requires to examine a high number of cause-effect relationship. The water distribution subsystem undoubtedly belongs to such systems. The FTA involves thinking back, which allows the identification of failure events that cause the occurrence of the Top Event. In the case of very large fault trees it is advisable to use computer methods (Tchorzewska-Cieslak & Boryczko, 2010) .

One of the FTA-based methods is the so-called bow-tie model that was used by Markowski and Kotynia (Markowski & Kotynia, 2011). It consists of a Fault Tree (FT) which identifies the causes of the undesired top event, and an Event Tree (ET) showing what is the consequence of such a release. So, this method encompasses the complete accident scenario using a bow-tie created by a Fault Tree and an Event Tree.

Pokorádi showed the adaptation of linear mathematical diagnostic modeling methodology for setting-up of Linear Fault Tree Sensitivity Model (LFTSM) (Pokorádi, 2011). The LFTSM is a modular approach tool that uses matrix-algebraic method based upon the mathematical diagnostic methodology of aircraft systems and gas turbine engines.

The logical method being presented in this paper is an adaptation of logical construction part of Fault Tree Analysis (FTA). This proposed method is named **LogTreeMM** - **Log**ical **Tree** of **M**athematical **M**odeling.

Pokorádi and Molnár showed the methodology of the Monte-Carlo Simulation and its applicability to investigate influences of fluid parameters to system losses by an easy pipeline system model. The (basically theoretical) obtained consequents and experiences can be used for investigation of parametrical uncertainties of the geothermal pipeline system, such as fluid characteristics indeterminations (Pokorádi & Molnár, 2011). This simulation model is practically used for demonstrating methodology of the proposed method.

The rest of this paper is organized as follows: Section 2 recalls the FTA methodology shortly and the logical tree method theoretically. Section 3 presents a possibility of use of the proposed method by a case study. Section 4 summarizes the paper, outlines the prospective scientific work of the Author.

2. Theoretical delineation

The proposed method is an adaptation of logical construction of FTA. The FTA is a systematic, deductive (top-down type) and probabilistic risk assessment tool, which shows the causal relations leading to a given undesired event. Bell Telephone Laboratories developed its concept at the beginning of the 1960s. It was adopted later and extensively applied by Boeing Company. FTA is one of several symbolic ”analytical logic techniques” found in operations research and in system reliability. The FTA is particularly useful for the analysis of complex technical systems where analysis of failure scenarios is a difficult process because it requires the examination of a high number of cause-effect relationships.

Fault Tree diagram displays on undesired state of the investigated system (Top Event) in terms of the states of its components (Basic Events). The FTA is a graphical design technique main result of which is a graph that has a dendritic structure.

The first step in a FTA is the selection of the Top Event that is a specific undesirable state or failure of a system.

After having analyzed the system so that we know all the causing effects we can construct the fault tree. Fault tree is based on ***AND*** and ***OR*** gates, which define the major characteristics of the fault tree.

The ***AND*** logical gate (Table 1) should be used if output event occurs only if all input events occur simultaneously. If the output event occurs if any of the input events occur, either alone or in any combinations, the ***OR*** logical gate (Table 1) should be used.

The Figure 1 shows a demonstrative Fault Tree. In the figure event B or C fail is Intermediate Event. The events A; B and C are Basic Events.

(After having the fault tree of the investigated undesired event, the probability of the Top Event can be analyzed depending of the probability of Basic Events. But it is not interesting in this study.)

The proposed method is analog with the above reviewed FTA technique. To construct LogTreeMM the following definitions are used:

A parameter can be known directly when

- its value is well-known (for example material characteristics);

or.

- it can be determined by direct measurement (for example internal diameter of a tube).

Let a variable be named Top Parameter if it should be determined but it is not used to calculate other one(s) in the investigated situation (is not an intermediate parameter).

Let a variable be called Intermediate Parameter if it has be known to calculate other one(s) but it cannot be known directly.

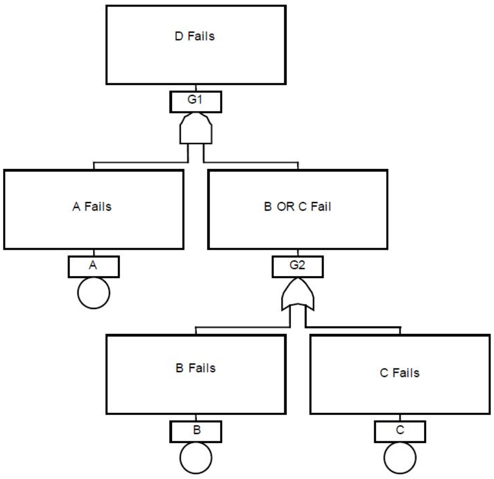
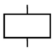


Figure 1. A Fault Tree (source: (Stamatelatos & Caraballo, 2002)).

Table 1
Symbols and Analogies between FTA and LogTreeMM

Symbol	FTA	LogTreeMM
	Top Event	Top Parameter
	Intermediate Event	Intermediate Parameter
	Basic Event	Basic Parameter
	***AND*** logical gate	
	***OR*** logical gate	

Let a variable be named Base Parameter if
- it is known directly;
or
- it cannot be determined by any equation (relation).

Let an ***AND*** logical gate (Table 1) be used if all of the independent variables should be known to calculate a dependent variable of the given equation (relation).

Let an ***OR*** logical gate (Table 1) be used if there are more than one equation (relation) on even terms to calculate the given dependent variable.

Table 1 demonstrates symbols used in FTA and LogTreeMM, as well as the analogies between their events and gates.

3. Case Study

To demonstrate step by step the logical tree method introduced above, we show a case study based on the simulation model presented in [3]. The study aimed the investigation of influences of fluid parameters to system losses in case of an easy pipeline system model. (Further on let the i-th logical gate be labeled by $/i/$.)

The illustrative system consisted of one lineal pipe and only one pipe fitting. The Δp pressure loss of this pipeline system as the Top Parameter can be determined by the equation

$$\Delta p = \Delta p_{cs} + \Delta p_{sz} \tag{3.1}$$

where:
Δp_{cs} pressure loss of linear pipe;
Δp_{sz} pressure loss of pipe fitting.

Thus two parameters (Δp_{cs} ***AND*** Δp_{sz}) should be known, which is shown by the ***AND*** gate of Figure 2. However, we do not know them directly. They are symbolized by rectangles, because they are Intermediate Elements of our investigation.

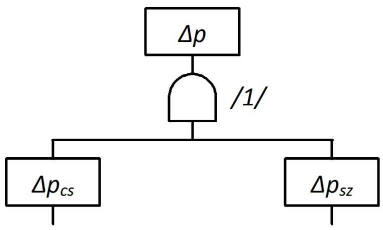


Figure 2. Logical Gate /1/ for Equation (3.1).

On the second level, the certain two structural elements (lineal pipe and pipe fitting) should be investigated. In the left branch, the pipe loss of linear pipe can be calculated by the equation

$$\Delta p_{cs} = \frac{\rho}{2} c^2 \frac{l}{d} \lambda \tag{3.2}$$

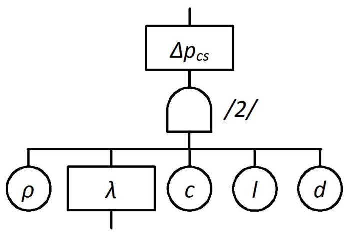


Figure 3. Logical Gate /2/ for Equation (3.2).

For that we should know fluid density ρ ***AND*** average fluid velocity c ***AND*** tube length l ***AND*** internal diameter d ***AND*** pipe loss coefficient λ. This logical sequence is shown by ***AND*** gate in Figure 3.

Except of the pipe lost coefficient (the rectangle shows that it is an Intermediate Parameter) all parameters can be determined directly (they are Basic Parameters), therefore they are shown by circles in Figure 3.

The pipe loss coefficient can be determined, depending only on Reynolds-number *Re* that cannot be determined directly (see Figure 4), by empirical equations in case of different Reynolds-number intervals:

$Re < 2320$:

$$\lambda = \frac{64}{Re} \tag{3.3}$$

$2320 < Re < 8\ 10^4$:

$$\lambda = \frac{0.316}{\sqrt[4]{Re}} \tag{3.4}$$

$2\ 10^4 < Re < 2\ 10^6$:

$$\lambda = 0.0054 + 0.396\ Re^{-0.3} \tag{3.5}$$

$10^5 < Re < 10^8$:

$$\lambda = 0.0032 + 0.211\ Re^{-0.337} \tag{3.6}$$

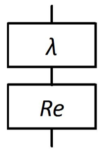


Figure 4. Connection of Equations (3.3) - (3.6).

For determination of the Reynolds-number *Re* the following equation should be used

$$Re = \frac{c\ d}{\nu} \tag{3.7}$$

For that we should know the average fluid velocity *c* ***AND*** the internal diameter *d* ***AND*** the kinematic viscosity of the fluid ν (they can be determined directly in other words, they are Basic Parameters).

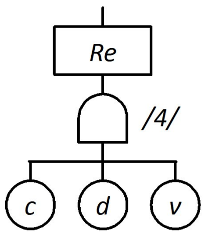


Figure 5. Logical Gate /4/ for Equation (3.7).

In the right branch of the second level, the loss pressure of pipe fitting can be determined by any of the following two equations

$$\Delta p_{sz} = \frac{\rho}{2} c^2 \xi \tag{3.8}$$

$$\Delta p_{sz} = \frac{\rho}{2} c^2 \frac{l_e}{d} \lambda \tag{3.9}$$

It is represented by the ***OR*** logical gate /3/ in Figure 6.

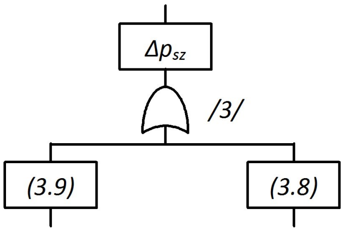


Figure 6. Logical Gate /3/.

In case of Equation (3.8), we need to know fluid density ρ ***AND*** average fluid velocity c ***AND*** pipe fitting loss coefficient ξ (see Figure 7).

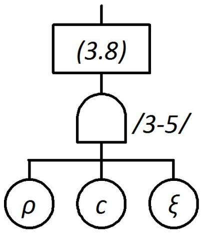


Figure 7. Logical Gate /3 − 5/ for Equation (3.8).

If the Equation (3.9) is used, the following parameters have to be known: fluid density ρ ***AND*** average fluid velocity c ***AND*** equivalent pipe length of pipe fitting l_e ***AND*** internal diameter of tube d ***AND*** pipe loss coefficient of tube λ (see Figure 8). The "equivalent pipe length" is length of given pipe, of which loss is equal to the loss of investigated pipe fitting in case of equal average fluid velocity.

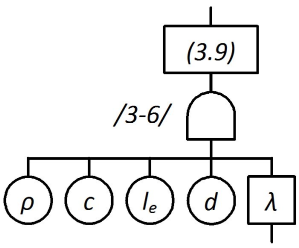


Figure 8. Logical Gate /3-6/ of Equation (3.9).

Assembling of the Figures 2 - 8, we get the logical tree of the illustrative model that is represented in Figure 9.

The sets of needed input parameters can be determined easily by investigating of the Logical Tree. For this purpose the subsets of the known parameters of the logical gates should be deducted first. In our case:

$$x_1 = \oslash \tag{3.10}$$

$$x_2 = \{\rho;\ c;\ l;\ d\} \tag{3.11}$$

$$x_3 = \oslash \tag{3.12}$$

$$x_4 = \{c;\ d;\ \nu\} \tag{3.13}$$

$$x_{3-5} = \{\rho;\ c;\ \xi\} \tag{3.14}$$

$$x_{3-6} = \{\rho;\ c;\ l_e;\ d\} \tag{3.15}$$

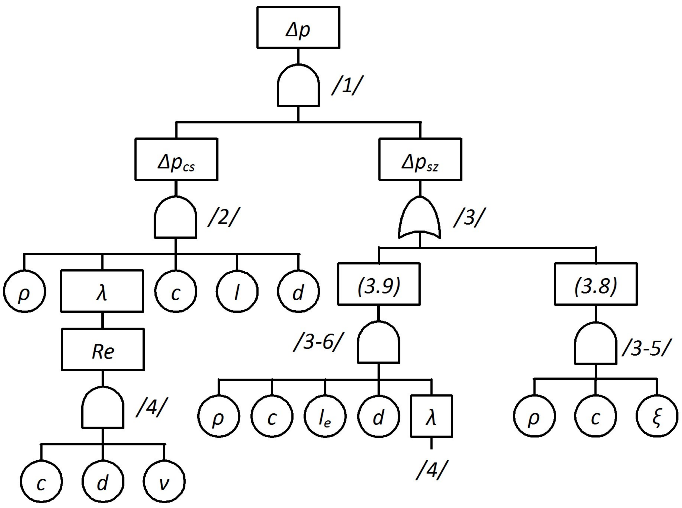


Figure 9. Logical Tree of the Case Study.

Knowing the subsets and the structure of the logical tree the possible sets of the needed parameters can be determined. The tree includes one two-input ***OR*** gate, therefore two sets of needed parameters can be identified:

$$x_A = x_2 \cup x_4 \cup x_{3-5} = \{\rho;\ c;\ l;\ d;\ \nu;\ \xi\} \tag{3.16}$$

$$x_A = x_2 \cup x_4 \cup x_{3-6} = \{\rho;\ c;\ l;\ d;\ \nu;\ l_e\} \tag{3.17}$$

It means that we should know either parameters from set x_A or from set x_B to set up and apply the mathematical model of the illustrative engineering problem. Fundamentally, the set x_A set was applied in the publication (Pokorádi & Molnár, 2011).

4. Conclusion

A logical tree method has been developed for the determination of possible sets of needed parameters for setting up of a mathematical model or solving an engineering calculation task. The method that named LogTreeMM is theoretically analogous with the Fault Tree Analysis used in system reliability assessment and quality management. The determined logical trees or their parts can be used as blocks to describe the required parameters in complex engineering calculation. In the education the LogTreeMM method can be used for developing of logical engineering thinking of students or pupils.

The Authors prospective scientific research related to this field of applied mathematics and engineering education includes the study of methodologies regarding technical system modeling and its decision making application in field of technical management.

References

Markowski, Adam S. and Agata Kotynia (2011). ”Bow-tie” model in layer of protection analysis. *Process Safety and Environmental Protection* **89**(4), 205 – 213.

Pokorádi, L. (2011). Sensitivity investigation of fault tree analysis with matrix-algebraic method. *Theory and Applications of Mathematics and Computer Science* **1**(1), 34–44.

Pokorádi, L. and B. Molnár (2011). Monte-Carlo simulation of the pipeline system to investigate water temperature's effects. *Polytechnical University of Bucharest. Scientific Bulletin. Series D: Mechanical Engineering* **73**(4), 223–236.

Stamatelatos, M. and J. Caraballo (2002). *Fault Tree Handbook with Aerospace Applications, Office of safety and mission assurance NASA headquarters*. NASA: Washington DC.

Tchorzewska-Cieslak, B. and K. Boryczko (2010). Relaxed LMI conditions for closed-loop fuzzy systems with tensor-product structure. *Engineering Applications of Artificial Intelligence* pp. 309–320.

Vesely, W.E., Goldberg F.F., Norman R. and Haasl D. (1987). *Fault tree handbook, Government Printing Office*. Government Printing Office: Washington DC.