Formation of the Optimal Investment Portfolio as a Precondition for the Bank’s Financial Security

A. S. Shapovalova, S. M. Shapovalova

Anna Shapovalova, Svetlana Shapovalova
Institute of Economics and Management, Vladimir Dahl
East-Ukrainian University, Severodonetsk, Ukraine

Abstract
This article analyses the definition of the bank’s financial security and investment activities. It describes a few types of models of bank’s risks management and the method CAPM, which is chosen for use. In support for the chosen CAPM method, we included the mathematical model that allows elaborating an optimal investment portfolio. The model stands at the basis of this method and a case study of one of Ukrainian banks.

Keywords: financial security, investment portfolio, risk, return on investment, variance, securities, covariance, yield.

Introduction
The experience of the developed states proves that the financial security of a bank is largely defined by approaches and forms of its investment activity.

The practice of investment activity in modern Ukrainian banks, even in the condition of high share of foreign capital, and therefore the possibility of leading world experience, unfortunately does not demonstrate the desired investment results. Such circumstances may be
explained by objective national realities. However, ignoring advanced techniques to build attractive investment portfolios is able to both deepen imperfections of investment activities of a bank and prevent effective risk management, threatening the financial security of the bank.

Risk taking is the basis of banking. Banks do well when risks taken by them are reasonable, under control and within their financial capabilities and competencies. Risk in investing activities occurs as a result of deviations of actual data on the evaluation of the current state and future development. These deviations can be both positive and negative. In the first case it goes about chance of receiving income, in the second – about the risk of loss. The relation between bank profitability and its risk (in simplified form) can be expressed as linear dependence.

However, if a bank seeks to ensure their financial security in general, and executing securities transactions, in particular, it is necessary to use risk management tools.

A lot of research has been dedicated to the issue of risk management and risk management of investment activity in particular [5, 2, 3], but those studies which examined risk measured it primarily in terms of integral component of the investment. If we consider risk as a factor that threatens the financial security of the banking activities, these studies have not dedicated attention to the management of investment risk for the banks financial security.

Therefore, the purpose of the article is to study the possibility and necessity of risk management that occurs in the process of investing to ensure the financial security of a bank, as well as demonstrating the theoretical and methodological steps of constructing the optimal investment portfolio of modern bank securities based on the approach of H. Markowitz, allowing to minimize the risk mentioned.

**Case Study**

The financial security of a bank is a state of that institution, which is characterized by a balance and resistance to external and internal threats (risks), its ability to achieve their goals and generate sufficient financial resources for sustainable development. Based on this definition, the bank's financial security can be ensured only in case of
the balance of its financial resources in the implementation of its activities (operating, financial or investment).

The article studies the investment activity of a bank as one of the most risky, but at the same time the most profitable banking activity. Therefore, the key to success in this business are effective risk management. There are many bank risk management methods, like: analysis and control gap, analysis and control durations.

The primary method of analysis is modeling. The main control methods are: neutralization of claims and liabilities; hedge of interest rate risk; the effective border; optimization of the portfolio structure by mathematical programming.

Given that almost all methods have been sufficiently studied and widely used in banking practice, the method of effective border has been studied in the article. This method is based on an application of defining the investment portfolio pricing model CAPM (Capital Asset Pricing Model) to the problem of interest rate risk analysis. Another name for this method is the method of average and deviation sample. Within the method there is considered the effectiveness of strategies depending on the associated risk. Under the strategy, in this case, it is necessary to understand the future cash flows generated by the current structure requirements and obligations and possible changes in the structure. As an indicator of the strategy efficiency there has been set mathematical expectation of the current or future value of cash flows, or other financial indicators, related to income and net worth. Standard deviation of expected performance is taken as an indicator of risk associated with a given strategy.

Prehistory occurrence of CAPM method was the discovery of H. Markowitz, who proposed a mathematical model of optimal investment portfolio, as well as methods of construction of such portfolios under certain conditions.

For the role of the facility to demonstrate the theoretical and methodological steps to construct the optimal modern bank securities investment portfolio, based on the approach of H. Markowitz, would best fit a bank that is actually engaged in investment activities (more or less successfully). Such bank is, for example, JSC "UkrSibbank".

H. Markowitz approach for JSC "UkrSibbank" will be treated as discrete, in which the beginning of the investment period is denoted as \( t = 0 \), and the end as \( t = 1 \). In this case, at the time \( t = 0 \), the investor must
make a decision to purchase specific financial instruments that will stay in its portfolio by the time \( t = 1 \).

As the portfolio of JSC "UkrSibbank" is a set of its particular variety, its decision is equivalent to selecting the optimal portfolio from the set of possible portfolios.

The approach of H. Markowitz as for portfolio choice implies that JSC "UkrSibbank" tries to solve two problems: to maximize expected yield for a given level of risk and minimize the uncertainty (risk) for a given level of expected yield.

As the investment portfolio is a collection of various assets (financial instruments), its yield can be calculated as follows:

\[
r_p = \frac{W_1 - W_0}{W_0} \quad (1)
\]

where: \( W_0 \) – is the aggregate purchase price of all assets included in the portfolio at time \( t = 0 \);

\( W_1 \) - the total market value of the assets at the time \( t = 1 \) and, in addition, the total cash income from ownership of the asset from the moment \( t = 0 \) until \( t = 1 \).

It is necessary to note that JSC "UkrSibbank" has to decide on which portfolio to buy at the time \( t = 0 \). In doing so, it does not know what the intended figure of profitability for most diverse alternative portfolios will be. Thus, JSC "UkrSibbank" must consider yield, associated with any of these portfolios, a random variable. These portfolios have their characteristics, one of them - the expected (or average) yield, and the other - the standard deviation.

With a portfolio of JSC "UkrSibbank" with a large number of assets in the future, it is expected that the number of companies which will be known any good news about will be equal to the number of companies which will be announced any bad news. This means that the more diversified the portfolio is, the less the unsystematic (own) risk will be. Thus, diversification significantly reduces unsystematic risk.

Systemic risk is manifested in another situation. Thus, with the number of assets included in the portfolio, systematic (market) risk converges to the mean for all pairs of assets included in the portfolio.

Thus, diversification allows averaging the systematic risk. This finding is important, as in the case of adverse or favorable economic outlook, most securities will not be paid or, respectively, paid. Despite
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the diversification of the portfolio, you can always expect that such market phenomena affect the profitability of the portfolio, especially as the market risk can not be eliminated through diversification.

For example, if you know only statistics on quoted yield securities, the further you can identify forward yield value by using various functions (logarithmic, linear or hyperbolic).

Expected values are not determined as the average, but as predicted extrapolation methods.

Estimated and actual value of the average return on assets for compiling a portfolio of securities of JSC “UkrSibbank” is shown in Figure no.1.

**Fig. no. 1.** Dynamics of six quoted yield securities (the forecast and actual values, drawn up according to the official website of JSC "UkrSibbank")
However, as noted above, any investment activity is related to risk. Therefore, there rises the question of assessing the risk level of JSC "UkrSibbank" securities.

To assess the risk of investments in securities there have been used indicators such as standard deviation and variance.

The standard deviation used as a risk assessment of the project, is also defined as a deviation from accepted values for trend forecasting.

Variance is a measure of deviation of the values of random variable distribution center. Larger values indicate greater dispersion deviation values of the random variable from distribution center.

Variance or distribution – is the mathematic expectation or mathematic expectation of a value raised to the second degree of deviation from its expected value (its expectation). So, variance is the measurement value of dispersion values of this variable, taking into account all its importance and their probability or weight.

Variance of discrete random variable is as follows:

\[ \sigma^2 = D(X) = E[(X - \mu)^2] = \sum_x (x - \mu)^2 p(x) \]  

(2)

Where

\[ \sigma = \sqrt{\sigma^2} \] is called the standard deviation value of its average value;

D – the operator of a random variable variance.

Let X, Y be two random variables defined on the same probability space.

Then their covariance is defined as follows:

\[ \text{cov}(X, Y) = E[(X - EX)(Y - EY)] \]  

(3)

assuming that all expectations E on the right side are defined. Yield indices values, values of variance, covariance risk and two portfolios are shown in table no.1.
Thus, if we know the portfolio yield and can calculate their riskiness, it becomes possible to determine an efficient set of portfolios (i.e. the set of not dominant portfolios that would let create the optimal structure). Under optimal investment portfolio structure in this case we can understand a set of financial instruments (securities) that would ensure such level of risk that would not create significant threats to the financial security of the bank.

The set of non-dominant portfolios, called effective solution, can be constructed by the solution of the general problem to minimize the risk, was first considered by H. Markowitz:

\[
\sigma^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j \sigma_{ij} \rightarrow \min
\]

where \(i\) – search index of the first pair of securities in the paired multiplication of the securities share and the covariance of securities included in the pair that are multiplied;

\(j\) – search index for the second in a pair of securities in doubles by multiplying the share of securities and the covariance of assets belonging to the couple multiplied;

\(n\) – number of securities in a portfolio;

\(\alpha\) – share of portfolio securities in fractions of a unit;

\(\sigma_{ij}\) – covariance of securities included in the pair multiplied when \(i \neq j\), of securities variance if \(i = j\), under two constraints. The first constraint captures the desired rate of yield, and the second constraint

<table>
<thead>
<tr>
<th>A couple of portfolios under consideration</th>
<th>Yield</th>
<th>Variance ((\sigma^2))</th>
<th>Covariance</th>
<th>(\sigma) (risk)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CJSC “Kyivoblenergo”</td>
<td>CJSC “First Ukrainian Bureau of Credit Histories”</td>
<td>59.24</td>
<td>57</td>
<td>-3.88</td>
</tr>
<tr>
<td>CJSC “First Ukrainian Bureau of Credit Histories”</td>
<td>JSC “Ukrainian Securities Depository”</td>
<td>48.73</td>
<td>27</td>
<td>11.54</td>
</tr>
<tr>
<td>JSC “Ukrainian Securities Depository”</td>
<td>CJSC “Kyivoblenergo”</td>
<td>23.26</td>
<td>32</td>
<td>6.41</td>
</tr>
</tbody>
</table>
normalizes the weights of the portfolio (without constraints on short position):

\[
\sum_{i} \alpha_i E(\eta_i) - \bar{E}(\alpha) = 0; \\
\sum_{i} \alpha_i - 1 = 0.
\]

The Lagrange objective function for the problem of minimizing risk at a fixed level of return is written as:

\[
L = \sum_{i}^{\sum_{j}} \alpha_i \sigma_{ij} + \lambda_1 \left( \sum_{j} \alpha_i E(\eta_j) - \bar{E}(\alpha) \right) + \lambda_2 \left( \sum_{i} \alpha_i - 1 \right) \tag{5}
\]

The portfolio that minimizes the risk is, if we put \( \partial L / \partial \alpha_i = \partial L / \partial \lambda_j = 0 \) for all \( i \) and shares for \( j = 1, 2 \). These first order conditions define a system of equations, linear weighting factor for portfolio and Lagrange multipliers and therefore it can be solved using matrix methods (with the possibility of using standard software packages). Thus, the objective function for the problem with three types of shares is written as:

\[
L = \alpha_1^2 \sigma_{11} + \alpha_2^2 \sigma_{22} + \alpha_3^2 \sigma_{33} + 2\alpha_1 \alpha_2 \sigma_{12} + 2\alpha_1 \alpha_3 \sigma_{13} + 2\alpha_2 \alpha_3 \sigma_{23} + \lambda_1 \left( \alpha_1 E_1 + \alpha_2 E_2 + \alpha_3 E_3 - \bar{E} \right) + \lambda_2 \left( \alpha_1 + \alpha_2 + \alpha_3 - 1 \right) \tag{6}
\]

After solving the Lagrange equation in the standard package of Microsoft Office Excel, you can find a set in which the coefficient of correlation are the number of securities listed in the investment portfolio of the company JSC "UkrSibbank". This solution determines the optimal portfolio of three companies’ securities that implements the required yield with minimum variance.

The structure of the portfolio JSC "UkrSibbank" can be represented graphically through fig. no. 2.

The initial value for determining the optimal structure of the investment portfolio is to search for a break-even point (without risky
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yield). The risk-free rate of yield makes 45.1%. The risk for such yield is the smallest – 3.885 (Table no. 2).

Table no. 2. Structure, risk and yield effective set portfolio JSC "UkrSibbank"

<table>
<thead>
<tr>
<th>Portfolio structure</th>
<th>CJSC &quot;Kyivoblenergo&quot;</th>
<th>CJSC &quot;First Ukrainian Bureau of Credit Histories&quot;</th>
<th>JSC &quot;Ukrainian Securities Depository&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Σ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Portfolio risk σ</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield</td>
<td>45.1</td>
<td>45.5</td>
<td>45.9</td>
</tr>
<tr>
<td></td>
<td>45.9</td>
<td>46.3</td>
<td>46.5</td>
</tr>
<tr>
<td></td>
<td>46.6</td>
<td>47.0</td>
<td>46.6</td>
</tr>
<tr>
<td></td>
<td>47.4</td>
<td>47.8</td>
<td>47.2</td>
</tr>
<tr>
<td></td>
<td>48.2</td>
<td>48.6</td>
<td>48.9</td>
</tr>
<tr>
<td></td>
<td>49.3</td>
<td>49.7</td>
<td>49.3</td>
</tr>
<tr>
<td></td>
<td>50.1</td>
<td>50.5</td>
<td>50.9</td>
</tr>
<tr>
<td></td>
<td>50.9</td>
<td>51.2</td>
<td>51.6</td>
</tr>
<tr>
<td></td>
<td>51.6</td>
<td>52.0</td>
<td>52.4</td>
</tr>
</tbody>
</table>

Fig. no. 2. Structure-yield of JSC "UkrSibbank" portfolio
Varying the desired yield, we can build an affordable and efficient set of portfolios JSC "UkrSibbank". Graphically determined without risk portfolio yield, affordable and effective set of securities JSC "UkrSibbank" is shown in the diagram (Fig. no. 3).

**Fig. no. 3.** Value of return and portfolio risk in different portfolios of JSC "UkrSibbank" securities

Following the outstanding securities effective set of JSC "UkrSibbank" the optimal structure can be chosen. Moreover, there is a choice as more risky and more profitable portfolio, and less risky, but also less profitable. The choice depends on the propensity management of JSC "UkrSibbank" to risk.

When choosing the optimal structure of the investment portfolio it should not be forgotten that risk management is not the only component that provides financial security of the bank. It should not be forgotten about the principle of comprehensiveness and consistency in providing financial security, because risk factors that threaten it may not only reduce the investment yield unpredictably, but also reduce the liquidity of the securities.
Conclusions

Financial security is a complex concept and its maintenance is only possible in complex analysis and control of the bank in order to prevent crises. Investment activity, thus, should be made only on the condition of financial security. It is therefore advisable to use different methods of managing investment risk of commercial banks, to which further research will be devoted to.

References