

An Investment Decision using Fuzzy Logic to select a Production Line

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Abstract

The paper presents a decision-making case study: the choice of a production line for natural juices, among 10 offers coming from 5 countries. 6 performance criteria are applied, some of them being fuzzy. Two solutions are provided: a conventional one, based on the affiliation degrees calculus and a fuzzy-interpolative one.

Keywords: decision-making, investments, fuzzy logic, fuzzy interpolative ADL matrix.

Introduction

The management decision-making is a difficult task because of the dimensions and complexity of the markets - raw materials, equipment, installations and business services. The selling companies have to understand the buyer's needs, resources, policies and buying procedures. The industrial buyer is an investor that faces a whole set of decisions in making a purchase. The number of decisions depends on the type of the buying situation. Making decision means to make a choice between more given possibilities. If the decision making describes an investment situation where the purchasing department reorders on a routine bases, we will call this “straight re-buy.” In this case the investor chooses the product that in the past gave him the higher buying and using satisfaction. “The modified re-buy” describes a purchasing situation where the buyer wants to modify the product specifications, prices or other terms [1, 2, 3, 4].

“The new task” faces a purchaser to buy a product or a service for the first time. The greater the cost and/or risk, the longer the list of decision participants and the greater their information seeking. The

number of decisions that the investor/buyer has to make is highest in the new-task situation.

Decision-making and the fuzzy theory

An industrial buyer is exposed to many influences when making a decision. Some marketers assume that the most important influence is economic: lowest price, best product or more services. Other marketers see buyers responding to personal factors such as favors, attention or risk avoidance. Industrial marketers must know their customers and adapt their tactics to individual, economical, organizational and environmental situations. All these factors contain different amounts of uncertainty and their weight in the final decision is also uncertain. Very often one can even consider them as perception based, affected by human subjective psychology [2, 5, 6]. The uncertainty always existed in human lives. The first mathematical tool designed to cope with the uncertainty is the probability theory. However the probability theory needs statistic data, which in many decision cases are missing – especially for the new task problems. This is why researchers quested for a new approach, able to cope when we can use only uncertain heuristics and perceptions. The first and basic answer to these quests is the fuzzy logic, due to Lotfi A. Zadeh [7, 8]. As shown in the literature, the theory of fuzzy sets and logic is able to represent linguistic modeled knowledge in computers, and to infer them in order to obtain decisions [9], etc.

The paper aims to illustrate a fuzzy based decision, using a conventional [5, 6] and a fuzzy-interpolative approach [10].

Case study

We will analyze the activity of a manager of a firm specialized in the production of 100% natural forest-fruit juice. The main activity of the firm is to collect forest fruits from all over Romania or to cultivate them in their own greenhouses, and to produce natural juices packed in Tetra-Pack. The juice production needs a new production line. The manager studied the market of the production lines for natural juices and find out that the highest quality of such plants are supplied by firms from: Italy, USA, Germany, Holland, Spain, France, Australia and Austria. Two of the 8 countries (USA and Germany) offer two types of products. The input variables:

The input variables

Table 1

C_1 = capacity (liters/hour)	C_4 = the payback time (years)
C_2 = the price (Euro)	C_5 = the maneuverability
C_3 = energy consumption (kW/h)	C_6 = firm's confidence degree

C_1 , C_2 , C_3 , and C_4 are quantitative while C_5 , and C_6 are qualitative variables. The variables are detailed in Table 2.

The detailed variables

Table 2

Firm	Country	C_1	C_2	C_3	C_4	C_5	C_6
V_1	Italy	55	100,500	50	3	medium	medium
V_2	USA	75	155,000	65	4	easy	low
V_3	USA	80	175,000	90	4	v. easy	high
V_4	Germany	90	180,000	100	5	easy	v. high
V_5	Germany	90	195,000	100	6	easy	medium
V_6	Holland	50	200,000	70	3	v. hard	low
V_7	France	60	185,000	60	7	v. hard	low
V_8	Spain	65	205,000	75	7	heavy	high
V_9	Australia	55	215,000	95	9	easy	high
V_{10}	Austria	50	165,000	95	9	heavy	v. low
-	k_j^*	2	2	1.66	1.34	2	1

An importance coefficient k_j^* , is attached to each variable. They are set with the *test of the universal specialist* (TSU). Two managers M1 and M2 and two engineers E1 and E2 make a top of the inputs according to their own expertise. Each place receives up to 6 points, according to its position. We impose $\sum k_j^* = 10$.

THE TSU TOP

Table 3

C_j	Managers		Engineers		Total	Top place	Points	k_j
	M1	M2	E1	E2				
C_1	3	5	4	6	18	I	6	2
C_2	4	6	3	5	18	I	6	2
C_3	6	4	5	1	16	II	5	1.66
C_4	1	2	2	3	8	III	4	1.32
C_5	5	3	6	4	18	I	6	2
C_6	2	1	1	2	6	IV	3	1

The affiliation degrees method

Suppose $V_i = \{V_1, V_2, \dots, V_i\}$ a multitude of alternatives concurring with a multitude of criteria $C_j = \{C_1, C_2, \dots, C_j\}$. V_1 is the alternative with the highest utility 1. V_0 is the alternative with the lowest utility 0. For example C_1 is a maximum criterion because we want the highest possible production capacity. The maximum C_1 will be set 1 in the matrix of the membership functions, the same as C_3 and C_4 . C_2 is a minimum criterion since we want the cheapest product and the lowest price will be set 0. C_5 and C_6 are qualitative criteria, and we associate them with the continuous interval $[0, 1]$:

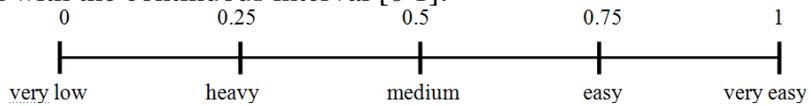


Fig. 1. Setting of the linguistic qualitative criteria for C_5 and C_6

Using the above scale and the points established for the six criteria, we shall build the matrix of distance degrees:

The matrix of distance degrees

Table 4

$C_j \backslash V_i$	C_1	C_2	C_3	C_4	C_5	C_6
V_1	0.38	0	0	0	0.5	0.5
V_2	0.17	0.35	0.23	0.25	0.75	0.25
V_3	0.11	0.42	0.44	0.25	1	0.75
V_4	0	0.44	0.5	0.4	0.75	1
V_5	0	0.48	0.5	0.5	0.75	0.5
V_6	0.44	0.49	0.28	0	1	0.25
V_7	0.33	0.45	0.16	0.57	1	0.25
V_8	0.27	0.51	0.93	0.57	0.25	0.75
V_9	0.38	0.53	0.47	0.66	0.75	0.75
V_{10}	0.44	0.39	0.47	0.66	0.25	0

The estimation of the distance degrees is different for the maximum and minimum criteria. For instance C_1 is a maximum criterion, so: x_1^* (distance degree for V_1 and C_1) = $1 - a_{ij} / a_{1j}$, where a_{ij} is the consequence of a variable V_i using C_j criterion. C_2 is a minimum criterion, so the calculus is inverse: $x_1^* = 1 - a_{1j} / a_{ij}$.

The matrix of the distance degrees x (coefficients of importance)

Table 5

$C_j \backslash V_i$	C_1	C_2	C_3	C_4	C_5	C_6
V_1	0.76	0	0	0	1	0.5
V_2	0.34	0.70	0.38	0.33	1.5	0.25
V_3	0.22	0.84	0.73	0.33	2	0.75
V_4	0	0.88	0.83	0.53	1.5	1
V_5	0	0.96	0.83	0.53	1.5	0.5
V_6	0.88	0.98	0.46	0	2	0.25
V_7	0.66	0.90	0.26	0.76	2	0.25
V_8	0.54	1.02	1.54	0.76	0.5	0.75
V_9	0.76	1.06	0.78	0.88	1.5	0.75
V_{10}	0.88	0.78	0.78	0.88	0.5	0

Using the table 5 values we can find the affiliation degree at the best variant that will be used to optimize the decisions. The affiliation degree is estimated by e^x and e^{-x} .

The matrix of affiliation degrees (e^x)

Table 6

$C_j \backslash V_i$	C_1	C_2	C_3	C_4	C_5	C_6	Σ	Σ/C_j
V_1	$e^{-0.76}$ =0.47	$e^0=1$	$e^0=1$	$e^0=1$	e^1 =0.37	$e^{-0.5}$ =0.61	4.45	0.74
V_2	$e^{-0.34}$ =0.71	$e^{-0.70}$ =0.5	$e^{-0.38}$ =0.68	$e^{-0.33}$ =0.72	$e^{-1.5}$ =0.22	$e^{-0.25}$ =0.78	3.61	0.60
V_3	$e^{-0.22}$ =0.8	$e^{-0.84}$ =0.43	$e^{-0.73}$ =0.48	$e^{-0.33}$ =0.72	e^{-2} =0.14	$e^{-0.75}$ =0.47	3.04	0.51
V_4	$e^0=1$	$e^{-0.88}$ =0.41	$e^{-0.83}$ =0.44	$e^{-0.53}$ =0.59	$e^{-1.5}$ =0.22	e^{-1} =0.37	3.03	0.51
V_5	$e^0=1$	$e^{-0.96}$ =0.38	$e^{-0.83}$ =0.44	$e^{-0.53}$ =0.51	$e^{-1.5}$ =0.22	$e^{-0.5}$ =0.61	3.16	0.53
V_6	$e^{-0.88}$ =0.41	$e^{-0.98}$ =0.38	$e^{-0.46}$ =0.63	$e^0=1$	e^{-2} =0.14	$e^{-0.25}$ =0.78	3.34	0.56
V_7	$e^{-0.66}$ =0.52	$e^{-0.90}$ =0.41	$e^{-0.26}$ =0.77	$e^{-0.76}$ =0.47	e^{-2} =0.14	$e^{-0.25}$ =0.78	3.09	0.52
V_8	$e^{-0.54}$ =0.58	$e^{-1.02}$ =0.36	$e^{-1.54}$ =0.21	$e^{-0.76}$ =0.47	$e^{-0.5}$ =0.61	$e^{-0.75}$ =0.47	2.70	0.45
V_9	$e^{-0.76}$ =0.77	$e^{-1.06}$ =0.35	$e^{-0.78}$ =0.46	$e^{-0.88}$ =0.41	$e^{-1.5}$ =0.22	$e^{-0.75}$ =0.47	2.38	0.4
V_{10}	$e^{-0.88}$ =0.41	$e^{-0.78}$ =0.46	$e^{-0.78}$ =0.46	$e^{-0.88}$ =0.41	$e^{-0.5}$ =0.61	$e^0=1$	3.35	0.56

The decision points the highest Σ/C_j value that is 0.74, so our manager will chose V_1 , the production line made in Italy.

The fuzzy decision tables

Although involving some qualitative criteria's, the above method is essentially numerical, using singletons for the modeling of the linguistic labels *very low*, *heavy*, *medium*, *high* and *very high*. This approach presents a minimum possible fuzziness, which is showing only in the heuristic setting of the singletons (see Fig. 1). A proper fuzzy approach replaces the matrixes filled with numbers with inference tables, filled with linguistic control rules.

In our case we have to draw a 6-D data base (six inputs), which will be fuzzyfied with piecewise automatically generated fuzzy partitions using triangular fuzzy sets. The automate generated fuzzy partitions are matching this classification problem, but this is not necessarily true in other kind of applications. We bound the variables' domains with the extreme values of Table 2. For instance the C1 input (capacity) will be defined on the [50 ... 90] segment. The fuzzy labels are *low*, *medium*, *high* for all the inputs and *very low*, *low*, *medium*, *high*, *very high* for the output *feasibility* $\in [0 \dots 1]$.

We will implement the decision-making system by the Matlab FIS toolkit (Fuzzy Inference System). The fuzzyfication of the six input variables and of the output is presentd in Fig. 2. The inference block and the rule viewer animation are shown in Fig. 3 and Fig. 4.

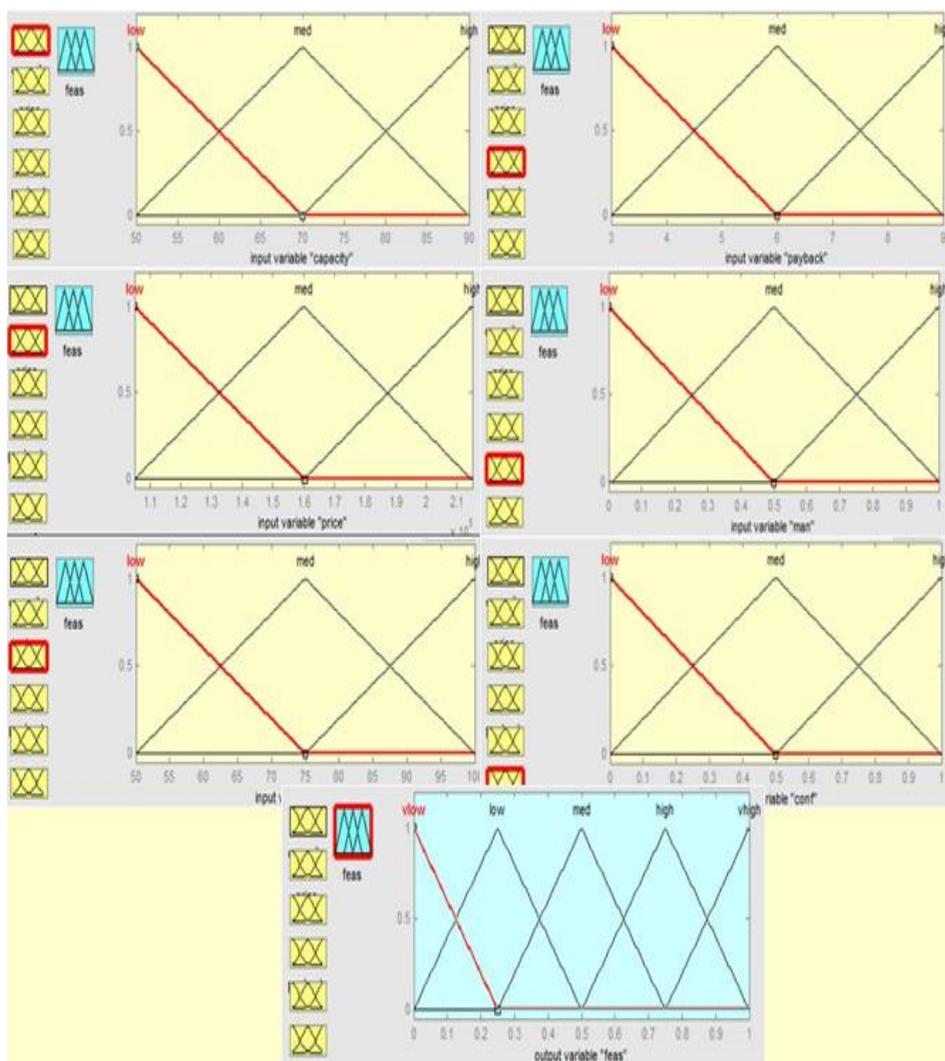


Fig. 2. The fuzzyfication of the six inputs and of the output *feasibility*

Fig. 3. The rule base

The screenshot displays a rule base editor with the following components:

- Rule List:** A scrollable list of 14 rules, each starting with "If" and followed by a series of conditions connected by "and", and ending with "then" and a consequent. The rules are:
 1. If (capacity is high) and (price is low) and (energy is low) and (payback is low) and (man is high) and (conf is high) then (feas is vhigh) (1)
 2. If (capacity is med) and (price is low) and (energy is low) and (payback is low) and (man is high) and (conf is high) then (feas is high) (1)
 3. If (capacity is high) and (price is med) and (energy is low) and (payback is low) and (man is high) and (conf is high) then (feas is high) (1)
 4. If (capacity is high) and (price is low) and (energy is med) and (payback is low) and (man is high) and (conf is high) then (feas is high) (1)
 5. If (capacity is high) and (price is low) and (energy is low) and (payback is med) and (man is high) and (conf is high) then (feas is high) (1)
 6. If (capacity is high) and (price is low) and (energy is low) and (payback is low) and (man is med) and (conf is high) then (feas is high) (1)
 7. If (capacity is high) and (price is low) and (energy is low) and (payback is low) and (man is high) and (conf is med) then (feas is high) (1)
 8. If (capacity is low) and (price is low) and (energy is low) and (payback is low) and (man is high) and (conf is high) then (feas is med) (1)
 9. If (capacity is high) and (price is high) and (energy is low) and (payback is low) and (man is high) and (conf is high) then (feas is med) (1)
 10. If (capacity is high) and (price is low) and (energy is high) and (payback is low) and (man is high) and (conf is high) then (feas is med) (1)
 11. If (capacity is high) and (price is low) and (energy is low) and (payback is high) and (man is high) and (conf is high) then (feas is med) (1)
 12. If (capacity is high) and (price is low) and (energy is low) and (payback is low) and (man is low) and (conf is high) then (feas is med) (1)
 13. If (capacity is high) and (price is low) and (energy is low) and (payback is low) and (man is high) and (conf is low) then (feas is med) (1)
 14. If (capacity is med) and (price is med) and (energy is low) and (payback is low) and (man is high) and (conf is high) then (feas is med) (1)
- Configuration Panel:** Located at the bottom, it includes:
 - Connection:** Radio buttons for "or" and "and", with "and" selected.
 - Weight:** A text input field containing the value "1".
 - Buttons:** "Delete rule", "Add rule", "Change rule", and navigation arrows ("<<", ">>").
 - Field Labels:** "capacity is", "price is", "energy is", "payback is", "man is", and "and" are positioned above their respective dropdown menus.
 - Dropdown Menus:** Each dropdown menu contains the options "low", "med", "high", and "none". The "high" option is selected in the "price is", "energy is", "payback is", and "man is" menus.
 - Checkboxes:** A "not" checkbox is present below each dropdown menu, and it is currently unchecked.

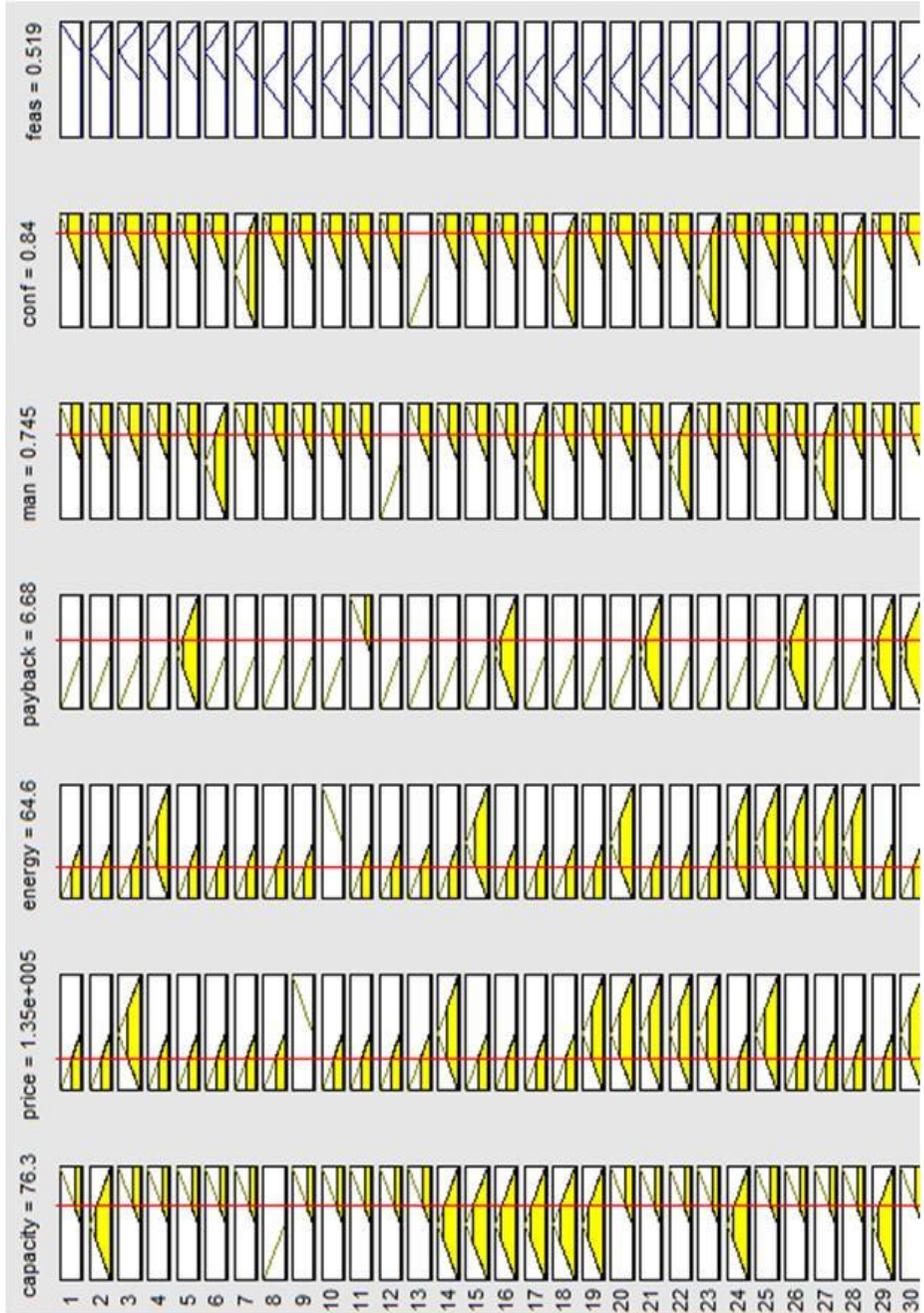


Fig. 4. The operation of the Simulink implementation

However, the $6^5 = 7776$ rules of the 6-D rule base is obviously a huge obstacle, although, as shown in Fig.3, we can significantly reduce the number of the rules. The “none” option of the inference dialog box allows us to write rules that are not involving all the six input variables.

A much more effective approach consists in clustering the input variables, with the purpose to reduce the dimension of the rule bases. One defines such way new internal variables, increasing the number of controllers, but dramatically decreasing the number of the rules. The most convenient internal variable is 2D, clearly representable by the McVicar-Whelan inference tables. Such a 2D table was applied in this field in ref. [11], concerning the fuzzy-interpretative version of the conventional ADL matrix.

The ADL matrix is a particular inference table that is often used for supporting strategic decisions [12]. The ADL Matrix infers a strategy for each of the different combinations of two input variables: *competitive position* and *industry maturity*, as shown in Table 7. The meaning of these variables is the following:

- *Competitive Position* CP - How strong is your strategic position?

- *Industry Maturity* IM - At what stage of its lifecycle is the industry?

In our case we will use this approach, clustering the input variables in three 2D decision tables: Technical level *Tech*($C_1 \times C_3$), Economical *Eco*($C_2 \times C_4$) and Subjective Perception *Subj*($C_4 \times C_5$). We want to reduce as much as possible the number of the linguistic labels so we use *Mamdani* controllers, *prod-sum* inferences and *Center of Gravity* defuzzyfications, a combination that maximize the sensitivity of the decision-making.

For instance, the *Tech* controller is presented in Fig. 5.

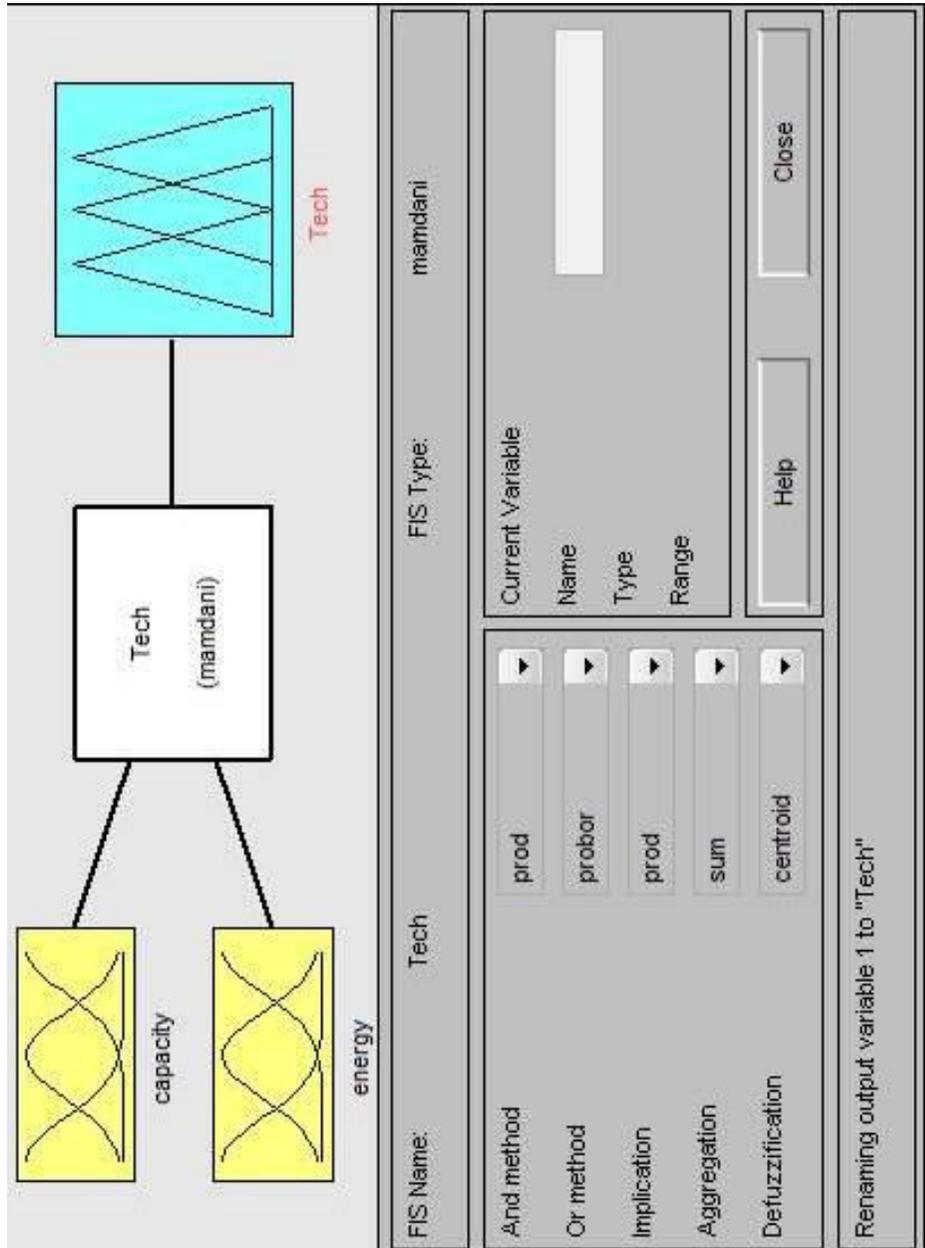


Fig. 5. The controller that is computing the *Tech* internal variable

Table 7

		Industry Maturity			
		Embryonic	Growth	Mature	Aging
C o m p e t i t i v e P o s i t i o n	Dominant	Y_{5,1} -Aggressive push for market share - Invest faster than market share dictates	Y_{5,2} - Maintain industry position and market share - Invest to sustain growth	Y_{5,3} - Maintain position, grow market share as the industry grows - Reinvest as necessary	Y_{5,4} - Maintain industry position - Reinvest as necessary
	Strong	Y_{4,1} -Aggressive push for market share - Look for ways to improve competitive advantage - Invest faster than market share dictates	Y_{4,2} -Aggressive push for market share - Look for ways to improve competitive advantage - Invest to increase growth and position	Y_{4,3} - Maintain position, grow market share as the industry grows - Reinvest as necessary	Y_{4,4} - Maintain industry position or cut expenditures to maximize profit (harvest) - Minimum reinvestment
	Favorable	Y_{3,1} - Moderate to aggressive push for market share - Look for ways to improve competitive advantage - Invest selectively	Y_{3,2} - Look for ways to improve competitive advantage and market share - Selectively invest to improve position	Y_{3,3} - Develop a niche or other strong differentiating factor and maintain it. - Minimum or selective reinvestment	Y_{3,4} - Cut expenditures to maximize profit (harvest) or plan a phased withdrawal - Minimum investment or look to get out of current investment
	Tenable	Y_{2,1} - Look for ways to improve industry position - Invest very selectively	Y_{2,2} - Develop a niche or other strong differentiating factor and maintain it - Invest selectively	Y_{2,3} - Develop a niche or other strong differentiating factor and maintain it or plan a phased withdrawal. - Selective reinvestment	Y_{2,4} - Phased withdrawal or abandon market - Get out of investments or divest
	Weak	Y_{1,1} - Decide if potential benefits outweigh costs, otherwise get out of market - Invest or divest	Y_{1,2} - Look for ways to improve share and position, or get out of the market - Invest or divest	Y_{1,3} - Look for ways to improve share and position or plan a phased withdrawal - Selectively invest or divest	Y_{1,4} - Abandon market - Divest

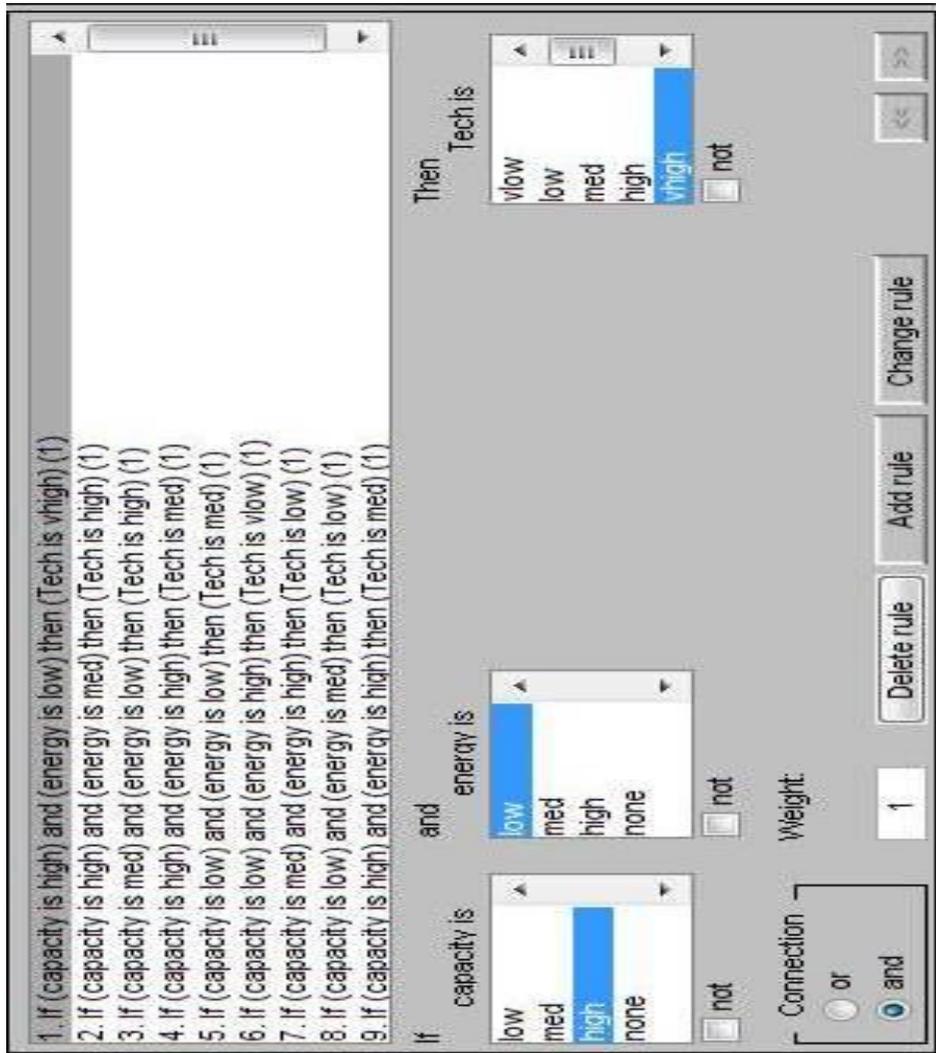


Fig. 6. The inference window, with the only nine rules

The rules are very easy to understand and to write:

- The best *Tech* ($Tech = 1$) is modeled by the rule “IF *cap* is *vhigh* AND *energy* is *vlow* THEN *Tech* is *vhigh*”.
- A medium *Eco* ($Eco = 0.5$) is pointed by three rules “IF *price* is *med* AND *payb* is *med* THEN *Eco* is *med*”, “IF *price* is *high* AND *payb* is *low* THEN

Eco is *med*” and “IF *price* is *low* AND *payb* is *high* THEN *Eco* is *med*”

- The worst *Subj* (*Subj* = 0) is pointed by the rule “IF *maint* is *vlow* AND *conf* is *vlow* THEN *Subj* is *vlow*”, etc.

We can use these three derived variables either in a final 3D decision table or as a weighted sum, taking into consideration the importance coefficients k_j .

$$Feas = (k_{Tech} *Tech + k_{Eco} *Eco + k_{Subj} *Subj) / (k_{Tech} + k_{Eco} + k_{Subj})$$

Setting by TUS the following values, $k_{Tech}=2$, $k_{Eco}=1.75$ and $k_{Subj}=1$, we eventually obtain the results of Table 8. The final choice, pointing the V_1 feasibility as the highest, is the same as in the previous method: $Feas(V_1) = 0.6811$.

Table 8

V_i	Feasibility	V_i	Feasibility
V_1	0.6811	V_6	0.3757
V_2	0.6783	V_7	0.3869
V_3	0.5913	V_8	0.3046
V_4	0.5620	V_9	0.2492
V_5	0.4675	V_{10}	0.1862

Improvements, like the implementation by look-up-tables (fuzzy-interpolative) [10] or the neural training, can be further provided. A fuzzy-interpolative system is a fuzzy system that can be equaled to a piecewise look-up table.

For instance, the look-up-table of the *Tech* variable (Fig. 6 rules) is:

Row (*Capacity*) = [50, 70, 90]

Column (*Energy*) = [50, 75, 100]

Table (*Tech*) = [0.5 0.25 0; 0.75 0.5 0.25; 1 0.75 0.5]

The *Tech* variable was fuzzyficated exactly as *Feas*, with five linguistic labels. The other variables, *Eco* and *Subj* were treated in the same way.

Conclusions

This paper presents two possible ways of using the fuzzy logic approach, in the managerial decision making field, for the case of a fruit juice production line purchase. A numerical affiliation degree matrix using singletons for the representations of the qualitative criteria is

compared with a fuzzy decision multi-dimensional table. The fuzzy-interpolative approach is more user friendly, thanks to the linguistic representation of the knowledge, and very cheap and effective, thanks to the interpolative implementation.

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